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Math 446
Third Midterm Exam
April 26, 2017

Show complete work to qualify for full credit. Perfect score **50 pts.**

[1] (12 pts) Compute the improper Riemann integral

$$I = \int_{-\infty}^{\infty} \frac{\cos(2x)}{x^2+9}\, dx.$$

Justify the steps in the computation.

[2] (8 = 4 + 4 pts) (i) Determine the number of zeros (counted with multiplicities) of

$$2z^2 - e^{z/2}$$

in the disk $|z| < 1$. Justify your answer.

(ii) Determine the number of zeros (counted with multiplicities) of

$$P(z) = z^4 + z^3 + 5z^2 + 7$$

in the left half-plane $\operatorname{Re} z < 0$. Justify your answer.

[3] (14 = 2 + 6 + 6 pts) (i) Define the Laplace transform of a function $u : [0, \infty) \to \mathbb{C}$.

(ii) Compute the residues at the two poles of the function

$$\frac{s^2 e^{st}}{(s^2+1)^2} \qquad (t \in \mathbb{R}).$$

(iii) Find the inverse Laplace transform $u = u(t)$ of the function

$$F(s) = \frac{s^2}{(s^2+1)^2}.$$

Justify your calculations.

[4] (8 pts) Find a linear fractional transformation that maps the real axis into the real axis and the imaginary axis into the circle $|z| = 1$.

[5] (8 = 5 + 3 pts) (i) State the Schwarz lemma.

(ii) Is there an invertible analytic function f mapping the unit disk $|z| < 1$ into itself and such that $f(0) = 0$ and $f\left(\frac{1}{2}\right) = \frac{1}{3}$?