Print name

# Math 446
## Second Midterm Exam
### March 27, 2017

**Show complete work to qualify for full credit.** Perfect score **50 pts.**

[1] (8 pts) Let $z_1, z_2, z_3$ be three distinct complex numbers and let

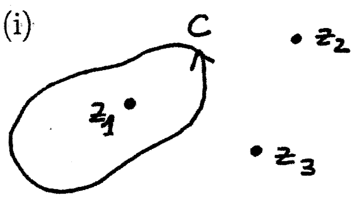
$$f(z) = \frac{1}{z - z_1} + \frac{1}{z - z_2} + \frac{1}{z - z_3}.$$

Indicate the value of the integral

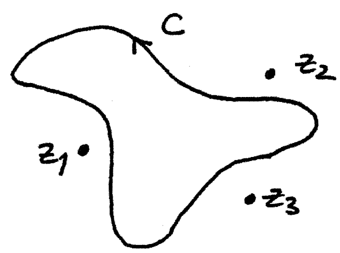
$$\int_C f(z)\, dz$$

in each of the following four situations:

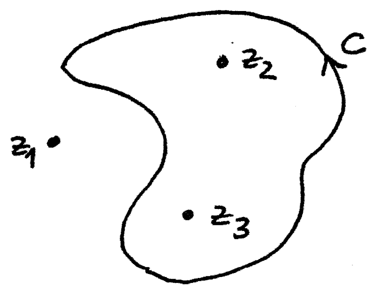
(i)



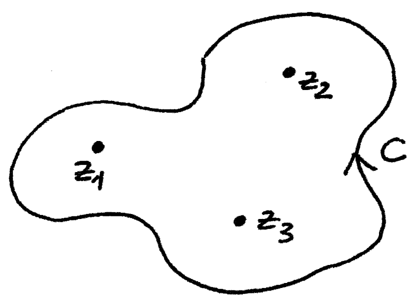

(ii)




(iii)




(iv)




[2] (6 pts.) State the Cauchy Integral Formula:
 (i) for $f(z_0)$ expressed as a contour integral;
 (ii) for $f^{(n)}(z_0)$ expressed as a contour integral ($n = 1, 2, 3, \ldots$).

[3] (6 pts) Let $f$ be an entire function such that $|f(z) - 2| \geq 1$ for every $z$. Prove that $f$ is a constant function.

[4] (10 pts) Find the two possible Laurent expansions centered at 0 of the function

$$f(z) = \frac{1}{z^2 - z}.$$

[5] (8 pts) Compute the coefficients $a_0, a_1, a_2$ and $a_3$ in the Taylor expansion

$$\frac{e^z}{1+z} = \sum_{n=0}^{\infty} a_n z^n = a_0 + a_1 z + a_2 z^2 + \cdots \qquad (|z| < 1).$$

[6] (12 pts) Consider the function $f(z) = \dfrac{1}{z^3(z-1)}$. Compute the following:

(i) $\operatorname*{Res}_{z=1} f(z)$; (ii) $\operatorname*{Res}_{z=0} f(z)$;

(iii) $\displaystyle\int_C f(z)\,dz$ where $C$ is the positively oriented circle $|z| = 2$.