Print name

Math 446

First Midterm Exam

February 17, 2017

Show complete work to qualify for full credit. Perfect score **50 pts.**

[1] (10 pts) Show that:

(a) (6 pts.) If $\operatorname{Re} z > 0 > \operatorname{Im} z$ and $w = z^2 - 1$, then $\operatorname{Im} w < 0$.

(b) (4 pts.) If $|z| < 1$ and $w = \dfrac{1+z}{1-z}$, then $\operatorname{Re} w > 0$.

[2] (12 pts.) Find the following values:

(a) (4 pts.) $\operatorname{Log}(-i)$;

(b) (4 pts.) $(-i)^i$;

(c) (4 pts.) $\exp\left[\pi\left(\dfrac{1+i}{\sqrt{2}}\right)^4\right]$.

[3] (10 pts) (a) (4 pts.) Describe all points z_0 in the complex plane where the function

$$f(z) = x^2 + y^2 + i(x^2 - y^2)$$

is differentiable, with $x = \operatorname{Re} z$, $y = \operatorname{Im} z$.

(b) (4 pts.) Show that the function

$$f(z) = \left(\frac{\bar{z}}{z}\right)^4$$

is not differentiable at any point z_0 in $\mathbb{C}$, $z_0 \neq 0$.

(c) (2 pts.) Show that $\lim_{z\to 0} \left(\dfrac{\bar{z}}{z}\right)^4$ does not exist.

[4] (10 pts) Find all complex roots z of the following two equations:

(a) (5 pts.) $\cos z = \sqrt{2}$;

(b) (5 pts.) $\sinh z = i$.

[5] (8 pts) Suppose that a function $f(z)$ is analytic at a point $z_0 = z(t_0)$ lying on a smooth arc $z = z(t)$ $(a \leq t \leq b)$. Show that if $w(t) = f[z(t)]$, then

$$w'(t) = f'[z(t)]z'(t)$$

when $t = t_0$.