Math 554
Homework 1
Due September 11, 2008, before class

Problem I Sets of measure zero

1. Show that a $C^1$ curve in the plane has measure zero.
2. Show that a countable reunion of sets of measure zero has measure zero

Problem II Measurable sets and functions. Use the definitions in Lecture 2.

1. If the functions $f$, $g$ are measurable show that $f+g$ and $\max\{f, g\}$ are also measurable.
2. If $A, B \subset \mathbb{R}^n$ are measurable then so are $A \cap B$, $A \cup B$ and the complement of $A$. (suggested) Generalize to a countable intersection and reunion.
3. Show that the cone $C = \{(x, y, z) \in \mathbb{R}^3 : z^2 \leq x^2 + y^2\}$ is a measurable set.
4. (suggested) Show that any bounded open set is measurable. Can you remove the boundedness condition?
5. Show that any closed set is measurable (you can use previous problem even if you did not do it).

Problem III Lebesque integral.

1. Show the elementary properties of Lebesque integral (use the definition in Lecture 2).
2. If $f$ is Riemann integrable on $[0, 1]$ then show that it is Lebesque integrable and the two integrals coincide. Is this still true if the improper Riemann integral on $[0, 1)$ exists and is finite (i.e. $\lim_{t \to 1^-} \int_0^t f(x)dx$ exists and it is finite)? What if the improper integrals on $[0, 1)$ of both $f$ and $|f|$ exists
3. (suggested) If you took a measure and integration theory class show that the definition of Lebesque integral from your class coincides with the one given in Lecture 2.