1. You have $1000, and a certain commodity presently sells for $2 per ounce. Suppose that after one week the commodity will sell either $1 or $4 per ounce, with these two possibilities being equally likely.

(a) If your objective is to maximize the expected amount of money that you possess at the end of the week, what strategy should you employ?

(b) If your objective is to maximize the expected amount of commodity that you possess at the end of the week, what strategy should you employ?

Solution: (a) To maximize the money, we should buy $0 \leq k \leq 500$ ounce of the commodity at the beginning of the week and sell it at the end. Then the random variable $X$ of the amount of money at the end of the week satisfies

$$P(X = 1000 - 2k + k) = P(X = 1000 - 2k + 4k) = 1/2.$$

So the expectation of it is

$$E[X] = \frac{1}{2}(1000 - 2k + k + 1000 - 2k + 4k) = 1000 + k/2.$$

To maximize it, we use all the money to buy 500 ounce of commodity at the beginning of the week and sell all of it at the end.

(b) To maximize the commodity, we should buy $0 \leq k \leq 500$ ounce of the commodity at the beginning of the week and buy it with the remaining money at the end. Then the random variable $Y$ of the amount of commodity at the end of the week satisfies

$$P(Y = k + (1000 - 2k)/1) = P(Y = k + (1000 - 2k)/4) = 1/2.$$

So the expectation of it is

$$E[Y] = \frac{1}{2}(k + (1000 - 2k)/1 + k + (1000 - 2k)/4) = 625 - k/4.$$

To maximize it, we should let $k = 0$ and use all the money to buy commodity at the end of the week.
2. If \( E[X] = 1 \) and \( Var(X) = 5 \), find

(a) \( E[(2 + X)^2] \);
(b) \( Var(4 + 3X) \).

\textbf{Solution:}  
(a) By the definition of the expectation, if the space of \( x \) is \( X \), then

\[ E[(2 + X)^2] = \sum_{x \in X} (2 + x)^2 p(x) \]
\[ = \sum_{x \in X} (4 + 4x + x^2) p(x) \]
\[ = 4 \sum_{x \in X} p(x) + 4 \sum_{x \in X} x p(x) + \sum_{x \in X} x^2 p(x) \]
\[ = 4 + 4E[X] + E[X^2]. \]

By using the identity

\[ Var(X) = E[X^2] - (E[X])^2, \]

we get

\[ E[X^2] = Var(X) + (E[X])^2 = 5 + 1^2 = 6, \]

so

\[ E[(2 + X)^2] = 4 + 4 \times 1 + 6 = 14. \]

(b) By using the identity

\[ Var(aY + b) = a^2 Var(Y), \]

we get

\[ Var(4 + 3X) = 3^2 Var(X) = 9 \times 5 = 45. \]