2. A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability 0.3, and his second appointment will lead independently to a sale with probability 0.6. Any sale is equally likely to be either for the deluxe model, which costs $1000, or the standard model, which costs $500. Determine the probability mass function of $X$, the total dollar value of all sales.

**Solution:** The range of $X$ is $\{0, 500, 1000, 1500, 2000\}$.

If we use the following abbreviations for events:

- $F$: the first appointment leads to a sale;
- $S$: the second appointment leads to a sale;
- $F_1$: getting a sale of 500 at the first appointment;
- $F_2$: getting a sale of 1000 at the first appointment;
- $S_1$: getting a sale of 500 at the second appointment;
- $S_2$: getting a sale of 1000 at the second appointment;

Then

\[
P(F^c) = 1 - P(F) = 1 - 0.3 = 0.7,
\]
\[
P(S^c) = 1 - P(S) = 1 - 0.6 = 0.4,
\]

\[F_i \subset F, \quad S_i \subset S \quad (i = 1, 2)\]

and

\[
P(F_1 \mid F) = P(F_2 \mid F) = P(S_1 \mid S) = P(S_2 \mid S) = 0.5.
\]

So for $i = 1, 2$ we have

\[
P(F_i) = P(F_i \mid F)P(F) + P(F_i \mid F^c)P(F^c) = 0.5 \times 0.3 + 0 = 0.15,
\]

and

\[
P(S_i) = P(S_i \mid S)P(S) + P(S_i \mid S^c)P(S^c) = 0.5 \times 0.6 + 0 = 0.3.
\]
Since two appointments are independent of each other,

\[ P(X = 0) = P(F^c \text{ and } S^c) \\
= P(F^c)P(S^c) \\
= (1 - .3)(1 - .6) \\
= .28; \]

\[ P(X = 500) = P((F_1 \text{ and } S^c) \text{ or } (F^c \text{ and } S_1)) \\
= P(F_1)P(S^c) + P(F^c)P(S_1) \\
= 0.15 \times 0.4 + 0.7 \times 0.3 \\
= .27; \]

\[ P(X = 1000) = P((F_2 \text{ and } S^c) \text{ or } (F_1 \text{ and } S_1) \text{ or } (F^c \text{ and } S_2)) \\
= P(F_2)P(S^c) + P(F_1)P(S_1) + P(F^c)P(S_2) \\
= 0.15 \times 0.4 + 0.15 \times 0.3 + 0.7 \times 0.3 \\
= .315; \]

\[ P(X = 1500) = P((F_1 \text{ and } S_2) \text{ or } (F_2 \text{ and } S_1)) \\
= P(F_1)P(S_2) + P(F_2)P(S_1) \\
= 2 \times 0.15 \times 0.3 \\
= 0.09 \]

and

\[ P(X = 2000) = P(F_2 \text{ and } S_2) \\
= P(F_2)P(S_2) \\
= 0.15 \times 0.3 \\
= .045. \]