1. An urn contains 6 white and 9 black balls. If 4 balls are to be randomly selected without replacement, what is the probability that the first 2 selected are white and the last 2 black?

**Solution.** If $W_i$ ($B_i$ resp.) is the event that the $i$th ball is white (black resp.), then by the multiplicative rule, we get

\[
P(W_1 \cap W_2 \cap B_3 \cap B_4) = P(W_1)P(W_2 | W_1)P(B_3 | W_1 \cap W_2)P(B_4 | W_1 \cap W_2 \cap B_3)
\]

\[
= \frac{6}{15} \times \frac{5}{14} \times \frac{9}{13} \times \frac{8}{12}
\]

\[
= \frac{3 \times 2}{7 \times 13}
\]

\[
= \frac{6}{91}.
\]
2. An urn contains 5 white and 10 black balls. A fair die is rolled and that number of balls is randomly chosen from the urn. What is the probability that all of the balls selected are white? What is the conditional probability that the die landed on 3 if all the balls selected are white?

**Solution:** Let \( R_i \) to be the event of getting an \( i \) while rolling the die and \( W \) to be the event that all the selected balls are white. Then

\[
P(R_i) = \frac{1}{6},
\]

\[
P(W \mid R_i) = \frac{\prod_{k=1}^{i} (5 - k + 1)}{\prod_{k=1}^{i} (15 - k + 1)} = \frac{5! \times (15 - i)!}{(5 - i)! \times 15!}
\]

and by Bayes' formula,

\[
P(W) = \sum_{i=1}^{6} P(W \mid R_i) P(R_i)
\]

\[
= \frac{1}{6} \frac{5!}{15!} \sum_{i=1}^{5} \frac{(15 - i)!}{(5 - i)!} = \frac{5! \times 10!}{6 \times 15!} \left[ \sum_{i=1}^{5} \binom{15}{i} \right]
\]

\[
= \frac{5! \times 10!}{6 \times 15!} \left[ \binom{15}{11} \right]
\]

\[
= \frac{5! \times 10! \times 15!}{6 \times 15! \times 11!} = \frac{5}{66}.
\]

By Bayes' formula again,

\[
P(R_3 \mid W) = \frac{P(W \mid R_3) P(R_3)}{\sum_{i=1}^{6} P(W \mid R_i) P(R_i)} = \frac{5 \times 4 \times 3}{15 \times 14 \times 13 \times 6} \times \frac{66}{5} = \frac{22}{455}.
\]