Summary

1. Probabilities

2. Random Variables

2.1 Discrete Random Variables

2.2 Continuous Random Variables

2.3 Jointly Distributed Random Variables

3. Properties of Expected Value (and Variance), Moment Generating Functions

4. Limit Theorems

4.1 Weak Law of Large Numbers

4.2 Markov's and Chebyshev's Inequalities

4.3 Central Limit Theorem
Random Combinatorial Analysis will not be directly checked but might play a role in computing probabilities.

1. Probabilities

Axiomatic Definition

Basic Properties:
- Probability of a complement event (set)
- Monotonicity
- Inclusion-exclusion principle
- Continuity with respect to increasing/decreasing sequences of sets (events)
  see § 2.6 or Lecture 9

Sample spaces having equally likely outcomes

Conditional Probabilities
- Definition
- Multiplication formula
- Bayesian formulas
- Independent events
For more details see Review for Calculus I, Lectures 4–8, Sections 2.1–2.6, 3.1–3.4
HW: 2–5

2. Random Variables

- general definition
- (cumulative) distribution function and its properties

See Lecture 9, Sections 4.1 & 4.10, HW: 6

2.1 Discrete Random Variables

- definition
- probability mass and distribution function
- expected value and variance
- expected value of a function of a r.v.
- Examples: Bernoulli & Binomial
  - Poisson
  - Geometric & Negative Binomial
  - Hypergeometric
2.2 Continuous Random Variables

- Definition, probability density and distribution function
- Expected value and variance
- Distribution of a function of C.R.V
- Examples: Uniform, normal, exponential & Gamma, Cauchy

For more details on 2.1, 2.2 see Review for Midterm II, Lectures 9-14, Sections 4.2-5.8, (no 4.3.4), 5.1-5.7 (no 5.5.1, 5.6.2, 5.6.4), HW: 6-10.

2.3 Jointly Distributed Random Variables

- Definition & properties of joint cumulative distribution function, marginal distribution
- Definition & properties of joint probability mass for d.n.v., marginal probability mass
- Definition of jointly continuous r.v.
Joint probability density and its properties, marginal density

- Conditional distribution:
  - definition of conditional probability mass function
  - definition of conditional probability density

- Sums of independent random variables

and applications to:

- Binomial
- Poisson
- Geometric
- Uniform
- Gamma

- Conditional distribution

See Lectures 15-17, Sections 6.1-6.7 (w/o 6.6)

HW: 10-12
3. Properties of Expected Value, Variance and Moment Generating Function

- Monotonicity of expected value
- Expected value of a sum of r.v.
- Covariance, correlation and applications to calculating the variance of a sum of r.v.
- Conditional expectation, definitions and applications to conditional variance, computing expected value and probabilities
- Moments of the number of events
- Moment generating function: defining properties and applications to:
  - Bernoulli & Binomial
  - Poisson
  - Geometric & Negative Binomial
  - Uniform
  - Normal
  - Exponential & Gamma
  - and Sums of independent r.v. of the type above except uniform.

See Lectures 18–21, Sections 7.1–7.7 (in 7.2.1, 7.2.2, 7.6), HW 12–14.
4. Limit Theorems

- Statement and proof of Weak Law of Large Numbers (either via characteristic function, moment generating function or Chebyshev’s Inequality)
- Statement and proof of Central Limit Theorem (either via characteristic or moment generating functions)
- Applications of Central Limit Theorem
- Statement, proofs and applications of Markov’s & Chebyshev’s inequalities
- Statement only of Strong Law of Large Numbers

See Lecture 22, Sections 8.1–8.4, HW 14.