Math 385, Section M1
Homework 12

The exercises are optional but do train you for the final. We’ll discuss them on Thursday.

**Problem I:** Find the Fourier series expansion of the solution of the string problem:

\[ u_{tt} = 4u_{xx}, \quad t > 0, \quad 0 < x < 6 \]

\[ u(t, 0) = 0, \quad u(t, 6) = 0, \quad t > 0 \]

\[ u(0, x) = \begin{cases} 
0, & 0 \leq x \leq 2, \\
x - 2, & 2 < x \leq 3, \\
4 - x, & 3 < x \leq 4, \\
0, & 4 < x \leq 6
\end{cases} \]

\[ u_t(0, x) = 0, \quad 0 \leq x \leq 6 \]

Use D’Alembert formula to find out what happens when the “triangular waves” reach the endpoints of the string.

**Extra Credit Problem:** Find the Fourier expansion of the solution of the problem. Pay careful attention to the first boundary (endpoint) condition:

\[ u_{tt} = 4u_{xx}, \quad t > 0, \quad 0 < x < 6 \]

\[ u_x(t, 0) = 0, \quad u(t, 6) = 0, \quad t > 0 \]

\[ u(0, x) = \begin{cases} 
0, & 0 \leq x \leq 2, \\
x - 2, & 2 < x \leq 3, \\
4 - x, & 3 < x \leq 4, \\
0, & 4 < x \leq 6
\end{cases} \]

\[ u_t(0, x) = 0, \quad 0 \leq x \leq 6 \]

Use D’Alembert **type** formula to find out what happens when the “triangular waves” reach the endpoints of the string.