Math 380, Section N1
Review Sheet for Midterm 2
October 29, 2006

1. **Second Order Partial Derivatives.** Know how to compute them when:
   - the function is given by a formula;
   - the function is a composition of two functions
   - the function is given implicitly, in other words \( F(x, y, z) = 0 \) gives \( z = z(x, y) \) and you need to find \( \frac{\partial^2 z}{\partial x \partial y} \) or other second order derivatives.

Suggested problems: see the Appendix

2. **Minima and Maxima for functions of several variables:**
   - Know how to compute absolute minima and maxima. Note that, when checking the boundary points, you might need to use Lagrange multipliers.
   - Know how to classify critical points using second order derivatives.
   - Know how to find minima and maxima of functions with side condition.


3. **Vector Differential Calculus:**
   - Know how to compute grad, div, curl of given scalar/vector fields given by concrete formulas.
   - Be able to deduce the following formulas
     \[
     \nabla (f + g) = \nabla f + \nabla g, \quad \nabla (fg) = f \nabla g + g \nabla f
     \]
     \[
     \nabla \cdot (u + v) = \nabla \cdot u + \nabla \cdot v, \quad \nabla \cdot (fu) = f \nabla \cdot u + \nabla f \cdot u
     \]
     \[
     \nabla \times (u + v) = \nabla \times u + \nabla \times v, \quad \nabla \times (fu) = f \nabla \times u + \nabla f \times u
     \]
     \[
     \nabla \cdot (u \times v) = v \cdot \nabla \times u - u \cdot \nabla \times v.
     \]
   - Memorize the above formulas and be able to manipulate them to deduce more complicated ones.

Suggested problems: Textbook pages 185-186 exercises: 4, 5 (do not find \( f \)), 6, 8 (recall that harmonic means \( \nabla^2 f = 0 \)), 11; and deduce the rest of the above formulas.

4. **Integral calculus for functions of several variables:**
   - know how to compute the integral via repeated integral and/or change of variables;
   - know how to use integrals to compute volumes of regions below surfaces.
5. Appendix:

(a) Consider

\[ f(x, y, z) = e^{xyz} + \sin\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{x}{z}\right). \]

Find all its second order derivatives at \( x = 1, \ y = 0, \ z = 2. \)

(b) The equation: \( \sin(xz) + y + x^3 - 1 = 0 \) defines \( x \) as a function of \( y \) and \( z \) near the point \( x = 1, \ y = 0, \ z = 0. \) Find:

\[
\frac{\partial^2 x}{\partial y^2}(0,0), \quad \frac{\partial^2 x}{\partial y \partial z}(0,0), \quad \frac{\partial^2 x}{\partial z \partial y}(0,0), \quad \frac{\partial^2 x}{\partial z^2}(0,0).
\]

(c) The equations

\[ e^u + xu - yv - 1 = 0, \quad e^v - xv + yu - 1 = 0, \]

determine \( u \) and \( v \) as functions of \( x \) and \( y \) near the point \( x = 0, \ y = 0, \ u = 0, \ v = 0. \) Find \( \frac{\partial^2 u}{\partial x \partial y} \) at \( x = 0, \ y = 0 \) and nearby.