Math 380, Section N1

Homework 1

Due August 31, 2006, before class

Consider the sets

\( S_1 = \) all points \((x, y)\), such that \(y > 0\);

\( S_2 = \) all points \((x, y)\), such that \(y \geq 0\);

\( S_3 = \) all points \((x, y)\), such that \(x^2 + y^2 < 1\);

\( S_4 = \) all points \((x, y)\), such that \(x^2 + y^2 \leq 1\);

\( S_5 = \) all points \((x, y)\), such that \(0 < x < 1, 0 < y < 1\);

\( S_6 = \) all points \((x, y)\), such that \(0 \leq x < 1, 0 < y < 1\).

**Problem 1.** Show that \(S_1\), \(S_3\) and \(S_5\) are open sets.

**Problem 2.** Show that \(S_2\) and \(S_4\) are closed sets.

**Problem 3.** Show that \(S_6\) is neither closed nor open.

**Problem 4.** Find an interior point of \(S_6\) and a boundary point for \(S_6\).

**Problem 5.** Characterize the boundary of \(S_1\), \(S_3\) and \(S_5\).

**Problem 6.** Which of the sets above are domains? Which are closed regions? Explain your answer.

**Problem 7.** (a) Show that a set is open if and only if all its points are interior points.

(b) Show that a set together with its boundary points form a closed set.

**Problem 8.** Textbook page 82, Exercise 2 parts c) and d). Do not use a computer for this problem. It defeats its purpose.

**Problem 9.** Textbook page 82, Exercise 3.