Math 285, Section P1
Homework 10
Will not be collected

Problem I In the following Euler type equations use the appropriate substitution \( t = \ln x \) to transform them in constant coefficient linear equations. Solve them using the characteristic equation and the method of undetermined coefficients:

\[
\begin{align*}
x^2 y'' - 4xy' + 6y &= x^3, \quad x > 0, \quad (1) \\
x^2 y'' - 3xy' + 4y &= x^4, \quad x > 0, \quad (2) \\
4x^2 y'' - 4xy' + 3y &= 8x^{4/3}, \quad x > 0, \quad (3) \\
x^2 y'' + xy' + y &= \ln x, \quad x > 0. \quad (4)
\end{align*}
\]

Problem II Study the method of variation of parameters (constants), see the end of Lecture 11 or the end of Section 3.5. Then solve (again!) exercises 53 and 54 from Section 3.5 in the textbook.

Problem III Recall how to find the solutions of a second order, linear, homogeneous equation when one solution is known, see the first part of Lecture 11. Combine it with the method of variation of parameters to find all solutions of the following equations when one solution \( y_1 \) of the homogeneous equation is given:

\[
\begin{align*}
x^2 y'' - x(x + 2)y' + (x + 2)y &= x^3, \quad x > 0, \quad y_1(x) = x, \quad (5) \\
(x + 1)y'' - (x + 2)y' + y &= (x + 1)^2, \quad x > -1, \quad y_1(x) = e^x, \quad (6) \\
(x^2 - 1)y'' - 2xy' + 2y &= x^2 - 1, \quad x > 1, \quad y_1 = x. \quad (7)
\end{align*}
\]