1. Let $A$ be a linear map of an $m$-dimensional vector space $V$ to itself, and let $\omega \in \Lambda^m(V^*)$. Compute $A^*\omega$.

2. A non-zero $k$-form $\phi \in \Lambda^k(V^*)$ is called decomposable if $\phi = \phi_1 \wedge \ldots \wedge \phi_k$ where the $\phi_j$ are all 1-forms. Show that if $\dim(V) < 4$ then every non-zero element of $\Lambda^2(V^*)$ is decomposable. Give a counterexample in dimension four.

3. Let $h : \mathbb{R}^1 \to S^1$ be $h(t) = (\cos t, \sin t)$. Show that if $\omega$ is any 1-form on $S^1$, then
   $$\int_{S^1} \omega = \int_0^{2\pi} h^* \omega.$$ 

4. A closed curve on a manifold $M$ is a map $\gamma : S^1 \to M$. If $\omega$ is a 1-form on $M$, define the line integral of $\omega$ around $\gamma$ by
   $$\oint_{\gamma} \omega = \int_{S^1} \gamma^* \omega.$$ 
   If $M = \mathbb{R}^k$, write $\oint_{\gamma} \omega$ explicitly in the coordinate expressions of $\gamma$ and $\omega$.

5. Prove that a 1-form $\omega$ on $S^1$ is the differential of a function iff $\int_{S^1} \omega = 0$. [Hint: “Only if” follows from Q4. Now let $h$ be as in Q3, and define a function $g$ on $\mathbb{R}$ by
   $$g(t) = \int_0^t h^* \omega.$$ 
   Show that if $\int_{S^1} \omega = 0$ then $g(t + 2\pi) = g(t)$. Therefore $g = f \circ h$ for some function $f$ on $S^1$. Check $df = \omega$.]

6. Let $\nu$ be any 1-form in $S^1$ with non-zero integral. Prove that if $\omega$ is any other 1-form, then there is a constant $c$ such that $\omega - c\nu = df$ for some function $f$ on $S^1$. 
