1. (a) Verify that the map
\[ t \mapsto \left( \frac{e^t + e^{-t}}{2}, \frac{e^t - e^{-t}}{2} \right) \]
is an embedding of \( \mathbb{R} \) into \( \mathbb{R}^2 \).
(b) Determine if the map \( F: \mathbb{R}^2 \to \mathbb{R}^3 \) given by
\[ F(x, y) = (x \cos y, x \sin y, x) \]
is an immersion.
(c) Check that the quotient map \( q: \mathbb{R}^{n+1} \setminus \{0\} \to \mathbb{R}P^n \) is a submersion.

2. If \( M \) is compact and \( N \) is connected show that every submersion \( F: M \to N \) is surjective.

3. Let \( M \) be a compact manifold and assume that \( \dim M = \dim N \). Let \( y \in N \) be a regular value of a smooth mapping \( F: M \to N \). Show that \( F^{-1}(y) \) is a finite set \( \{x_1, \ldots, x_k\} \). Prove that there exists a neighborhood \( V \) of \( y \) in \( N \) such that \( F^{-1}(V) \) is a disjoint union \( U_1 \cup \ldots \cup U_k \) where each \( U_j \) is an open neighborhood of \( x_j \) and \( F|_{U_j} \) is a diffeomorphism onto \( V \).

4. Show that the image of \( S^2 \subset \mathbb{R}^3 \) under the map
\[ F: (x, y, z) \mapsto (x^2, y^2, z^2, \sqrt{2}yz, \sqrt{2}zx, \sqrt{2}xy) \]
is a submanifold of \( \mathbb{R}^6 \). Hint: consider \( F \circ q \) for the quotient map \( q: \mathbb{R}^3 \setminus \{0\} \to \mathbb{R}P^2 \).