Math 423  Differential Geometry  Fall 2006

Homework 4: The Main Theorem of Space Curves

Due Thursday Sept. 28

1. Section 3.5 #3
2. Section 3.5 #6
3. Section 3.5 #7

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**Bonus Questions** for those taking the course for four credits or for those who want more of a challenge.

4. The Frenet frame is not the only useful way to frame a curve. Let \( \alpha: I \to \mathbb{R}^3 \) be a unit speed curve and consider a frame \((T(t), E_1(t), E_2(t))\) along \(\alpha\) which is positively oriented and satisfies

\[
E'_1(t) = -k_1(t)T(t) \quad \text{and} \quad E'_2(t) = -k_2(t)T(t)
\]

for some functions \(k_1(t)\) and \(k_2(t)\).

(a) Prove that two curves with the same \(k_1(t)\) and \(k_2(t)\) are congruent.

(b) Assuming \(\kappa(t) > 0\), express \(k_1(t)\) and \(k_2(t)\) in terms of \(\kappa(t)\) and \(\theta(t)\) where \(\theta(t)\) is defined by the equation

\[
N(t) = \cos \theta(t)E_1(t) + \sin \theta(t)E_2(t).
\]

(c) Express \(\kappa\) and \(\tau\) in terms of \(k_1\) and \(k_2\).

(d) Show that \(T' = k_1E_1 + k_2E_2\).

(e) Show that \(k_1(t)\) and \(k_2(t)\) are constant iff \(\alpha\) is part of a circle. What is the relation of the radius of the circle and \(k_1\) and \(k_2\)?

(f) Show that \(\alpha\) is a plane curve iff \(k_1(t)/k_2(t)\) is constant.