Worksheet #2
Math 221

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

1. The derivative of \(x^2\).

   (a) Explain why the slope of the secant line for \(y = x^2\) through \(x = 3\) and \(x = 3 + h\) (for any number \(h \neq 0\)) is
   \[
   \frac{(3 + h)^2 - 9}{h}
   \]

   (b) Show that \(\frac{(3+h)^2-9}{h} = 6h + h^2\).

   (c) Explain why \(\frac{6h + h^2}{h} \neq 6 + h\) and then explain why
   \[
   \lim_{h \to 0} \frac{6h + h^2}{h} = \lim_{h \to 0} 6 + h = 6
   \]

   (d) Redo this exercise replacing 3 by \(a\) to show that \(\frac{d}{dx} x^2|_a = 2a^1\).

2. Sketch the graph of a function \(f\) with the properties that
   \[
   \lim_{x \to 2^-} f(x) = 5 \quad f(2) = 6 \quad \lim_{x \to 2^+} f(x) = 7
   \]

3. Give a “real world” example of a function (over time) such that the left hand limit to some time exists but is not equal to the value of the function at that time.