Math 518 Differentiable Manifolds I

Assignment 4, Due Thursday October 21

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1. Let X be a smooth vector field on S^1 which vanishes at $p \in S^1$. Choose a chart (U, φ) around p with $\varphi(p) = 0$. In the corresponding local coordinates X has the form
$$f(x)\frac{\partial}{\partial x}|_x$$
where f is a smooth function on $\varphi(U)$ and $f(0) = 0$.

   (a) Show that the constant $\frac{\partial f}{\partial x}|_{x=0}$ does not depend on the choice of the chart (U, φ).

   (b) The zero p of X is called *nondegenerate* if the constant $\frac{\partial f}{\partial x}|_{x=0}$ is nonzero. Show that if X has only nondegenerate zeros then it must have finitely many zeros.

2. (a) Prove that the group
$$SO(3, \mathbb{R}) = \{A \in M(3, \mathbb{R}) \mid \det(A) = 1,\ AA^T = I_3\}$$
is a smooth submanifold of $M(3, \mathbb{R})$.

   (b) Consider the subset of $T\mathbb{R}^3 \cong \mathbb{R}^3 \times \mathbb{R}^3$ consisting of tangent vectors to the unit sphere S^2 which have unit length. Prove that this subset is a smooth submanifold.

   (c) Show that the intersection of the sphere $|z_1|^2 + |z_2|^2 + |z_3|^2 = 1$ in $\mathbb{C}^3$ with the complex cone $z_1^2 + z_2^2 + z_3^2 = 1$ is a smooth submanifold.

   (d) (Just something to consider.) Are any of these manifolds diffeomorphic to each other?

Problems from the text: 3-5, 7-2, 8-1, 8-2.