Math 518 Differentiable Manifolds I

Assignment 3, Due Tuesday October 12

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1. Consider the 3-sphere $S^3 \subset \mathbb{R}^4$. Using the isomorphism $\mathbb{R}^4 \cong \mathbb{C}^2$ we obtain the inclusion $\imath\colon S^3 \to \mathbb{C}^2 \setminus \{0\}$. Composing with the projection map $q\colon \mathbb{C}^2 \setminus \{0\} \to \mathbb{C}P^1$ we then get the map

$$p = q \circ \imath\colon S^3 \to \mathbb{C}P^1$$

which is known as the *Hopf fibration*. Using our charts for S^3 and $\mathbb{C}P^1$ verify that the map p is smooth.

2. A vector field $Y\colon M \to TM$ is said to vanish at p if $Y(p)$ is the trivial derivation at p. Equivalently, Y vanishes at p if in local coordinates we have

$$Y(p) = \sum_i 0 \frac{\partial}{\partial x_i}|_p.$$

Using the stereographic charts for S^2 defined in class, construct a smooth vector field on S^2 which vanishes at exactly two points, and another one which vanishes at exacly one point.

3. If K is a submanifold of L and L is a submanifold of M, is K necessarily a submanifold of M? Justify your answer with either a proof or a counterexample.

Problems from the text: 3-1, 3-2, 3-3.