1. Let $\alpha$ be a 2-form on $\mathbb{R}^3$ given by
\[ \alpha = 2xy \, dx \wedge dy - (x^2 + y + 1) \, dy \wedge dz. \]
(a) Compute $d\alpha$.
(b) Let $F: \mathbb{R}^2 \to \mathbb{R}^3$ be the smooth mapping defined by
\[ F(u, v) = (u + v, u^2 + 1, 3uv - 4). \]
Compute $F^*\alpha$.
(c) Let $\beta = dx + 2 \, dy + 3 \, dz$. Compute $\alpha \wedge \beta$ and $F^*(\alpha \wedge \beta)$.

2. Consider the following two vector fields on $\mathbb{R}^3$:
\[ X = 2xy \frac{\partial}{\partial x} + (x^2 + z^2) \frac{\partial}{\partial z}, \]
\[ Y = e^{xy} \frac{\partial}{\partial x} + \cos(2xz) \frac{\partial}{\partial y}. \]
Compute the vector field $[X, Y]$.

3. Let $\alpha$ be a 1-form on a 3-dimensional manifold $M$. Prove that $d(d\alpha) = 0$.

4. Let $\omega$ be an $r$-form on a manifold $M$ with $r \geq 2$. Suppose that the tangent vectors $V_1, \ldots, V_r \in T_pM$ are linearly dependent. Prove that
\[ \omega(p)(V_1, \ldots, V_r) = 0. \]

5. Let $X, Y$ and $Z$ be smooth vector fields on a manifold $M$. Prove the Jacobi Identity
\[ [[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y] = 0. \]