Math 416 Additional Practice Questions for Exam 2

Here are some basic questions that you should be able to answer without help or reference if you are prepared for EXAM 2.

(1) Let $A$ be an $n \times n$ matrix.
   (a) Write down a formula for the determinant of $A$.
   (b) Use your formula to prove that if $B$ is obtained from $A$ by multiplying its second row by the scalar $k$, then $\det(B) = k \det(A)$.
   (c) Prove that $\det(kA) = k^n \det(A)$.
   (d) Let $E$ be an $n \times n$ elementary matrix of type (i), (ii) or (iii). In all three cases write down the relationship between $\det(A)$ and $\det(EA)$.

(2) Suppose $\dim(V) = n$ and $T \in \mathcal{L}(V, V)$.
   (a) Define the characteristic polynomial of $T$. Define what it means for it to split.
   (b) Define what it means for $\lambda$ to be an eigenvalue of $T$.
   (c) Define the algebraic multiplicity of $\lambda$. Write down a procedure to compute it. Prove that it is always at least 1.
   (d) Define the geometric multiplicity of $\lambda$. Write down a procedure to compute it. Prove that it is always at least 1.
   (e) Suppose $\beta$ and $\beta'$ are both bases for $V$. Write down the change-of-basis formula relating $[T]_\beta$ and $[T]_{\beta'}$. Use this to prove that the characteristic polynomial of $T$ does not depend on the basis chosen to define it.
   (f) Define what it means for a subspace $W$ of $V$ to be $T$-invariant. Prove that $R(T)$ is $T$-invariant.