MATH 225 REVIEW PROBLEMS FOR CHAPTER 6

Suggested problems from the text:
Section 6.1 ; 1–8, 13–18, 19, 20, 23, 24.
Section 6.2 ; 7–16, 23, 24 (a) (d), 26.
Section 6.3 ; 3–6, 9–12, 15, 16.
Section 6.5 ; 1–6, 7–9, 11, 13, 19, 21, 23, 25–28.

Extra Practice Problems:
(1) Let \( u \) and \( v \) be vectors in \( \mathbb{R}^n \). Show that \( \|u + v\|^2 = \|u\|^2 + \|v\|^2 \) if and only if \( u \cdot v = 0 \).

(2) (a) Define \( \text{proj}_v y \), the projection of the vector \( y \) onto the line determined by \( v \).
    (b) Show that \( \text{proj}_v y \) is the vector on the line determined by \( v \) which is closest to \( y \).
    (c) Compute \( \text{proj}_v y \) for \( y = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \) and \( v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \).

(3) (a) Define what it means for a set of vectors \( \{v_1, v_2, \ldots, v_p\} \) to be orthogonal.
    (b) Show that if the set \( \{v_1, v_2, \ldots, v_p\} \) is orthogonal then it is also linearly independent.

(4) (a) Define \( \text{proj}_W y \), the projection of the vector \( y \) onto the subspace \( W \).
    (b) Show that \( \text{proj}_W y \) is the vector in \( W \) which is closest to \( y \).
    (c) Compute \( \text{proj}_W y \) for \( y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \) and

\[
W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}.\]
(d) For $y$ and $W$ as in part (c), find the minimum value of $\|y - w\|$ for all $w$ in $W$.

(5) Find a least squares solution for the following linear systems:

(a)

\[
\begin{align*}
    x_1 + x_2 &= 4 \\
    x_1 &= 1 \\
    x_2 &= 1
\end{align*}
\]

(b)

\[
\begin{align*}
    x_1 + x_2 &= 2 \\
    x_1 - x_2 &= 2 \\
    x_1 + 2x_2 &= 1.
\end{align*}
\]