MATH 225 REVIEW PROBLEMS FOR MIDTERM 1

Details: Midterm 1 will take place in class on Tuesday Sept. 23. It will cover the material from sections 1.1 – 1.7. (For section 1.6 you are only responsible for balancing chemical equations.)

Suggested problems from the text, pages 102-104:

- Question 1 minus x, y, and z.
- Questions 2 – 19.

Extra Practice Problems:

(1) (a) If $A$ is an $m \times n$ matrix and $x$ is an $n$-vector, define $Ax$ in terms of the column vectors of $A$ and the entries of $x$.
   (b) Using your formula from part (a), prove that $A(cx) = c(Ax)$ for any scalar $c$.

(2) For what values of $h$ and $k$ is the system of linear equations with the augmented matrix

\[
\begin{bmatrix}
1 & h & 1 \\
5 & 3 & k
\end{bmatrix}
\]

inconsistent?

(3) Consider the matrices

\[
A_1 = \begin{bmatrix}
1 & 1 & 1 & 1 \\
2 & 1 & 1 & 1 \\
3 & 2 & 2 & 0
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
1 & 1 & 1 & 1 \\
2 & 1 & 1 & 1 \\
3 & 2 & 2 & 0
\end{bmatrix}.
\]

Solve the equations $A_1x = b$ and $A_2x = b$ for

(a) $b = \begin{bmatrix}
1 \\
2 \\
3
\end{bmatrix}$,  
(b) $b = \begin{bmatrix}
1 \\
1 \\
3
\end{bmatrix}$,  
(c) $b = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}$.

(4) Give a geometric description of each of the solution sets from problem (3).

(5) (a) Define what it means for vectors $v_1, \ldots, v_p$ in $\mathbb{R}^n$ to be linearly dependent.
(b) Determine whether the following vectors are linearly dependent

\[
v_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3 \end{bmatrix}.
\]