Math 231 ABCE. Spring, 2012. Worksheet 5. 02/01/2012 and 02/02/2012

1. Write out the form of the partial fraction decomposition of each function. **Do not evaluate the numerical values of coefficients. Leave them in the form \( A, B, C, \cdots \) or \( Ax + B, Cx + D, \cdots \)**

   For example, your answer might include the term \( \frac{A}{x-1} \).

   (a) \( \frac{8x^2 + 11}{(x^3 - 3x^2 + 2x)(x+5)} = \)

   (b) \( \frac{3x^3 - 8}{x^3 + 2x^2 + x} = \)

2. Use partial fractions to evaluate the following integrals.

   (a) \( \int \frac{1}{x^2 - 3x + 2} \, dx \)

   (b) \( \int \frac{3x - 8}{x^3 + x} \, dx \)
3. Use polynomial division and partial fractions to evaluate.

\[ \int \frac{x^3 + 1}{x^2 - 3x + 2} \, dx. \]

4. This problem presents an alternate way to integrate \( \sec \theta \) (remember that the trick which we used in class involved multiplying top and bottom by \( \sec \theta + \tan \theta \)).

(a) Use the identity \( \sec \theta = \frac{\cos \theta}{1 - \sin^2 \theta} \) and make a \( u \)-substitution to get the integral of a rational function.

(b) Integrate the rational function by partial fractions to get

\[ \int \sec \theta \, d\theta = \ln \left| \frac{\sqrt{1 + \sin \theta}}{1 - \sin \theta} \right| + C. \]

Then show why this is equivalent to the more familiar answer: \( \ln |\sec \theta + \tan \theta| + C. \)

\[^{1} \text{HINT: Inside the radical multiply top and bottom by } 1 + \sin(\theta). \]