1. Consider the curve $x(t) = 2t^2$, $y(t) = t^3 - 4t$

(a) Find all values of $t$ for which the tangent line is horizontal or is vertical.

(b) Use $x'(t)$ and $y'(t)$ together with intercepts and behavior as $t \to \pm\infty$ to sketch the curve.

(c) Find the slope of the two tangent lines at $(8, 0)$.

(e) Set up but do not evaluate an integral for the area under the portion of the curve from $t = 2$ to $t = 3$ and above the $x$-axis.
2. The Scrambler! The hypocycloid is traced by a circle of radius $b$ which rolls without slipping around the inside of a larger circle of radius $a$. This curve is traced by the famous carnival ride “The Scrambler.” Assume that $a = 12 \text{ m}$ and $b = 3 \text{ m}$ (as shown below).

The parametric equations for this hypocycloid curve are

$$x(\theta) = 9 \cos(\theta) + 3 \cos(3\theta), \quad y(\theta) = 9 \sin(\theta) - 3 \sin(3\theta).$$

($\theta$ is the angle between the positive $x$-axis and a ray through the center of the small circle.)

(a) Find the vector of first derivatives $(x'(\theta), y'(\theta))$.

(b) Evaluate this vector at each value of $\theta$: (i) $\theta = 0$. (ii) $\theta = \pi/4$. (iii) $\theta = \pi/2$.

What does this tell you about the tangent lines at these three points?

(c) The length of a parametric curve from $t = a$ to $t = b$ is $s = \int_a^b \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt$.

Write an integral for the length of the curve through one complete revolution, $\theta = 0$ to $\theta = 2\pi$. Simplify as much as possible.