1. Recall the binomial series:

\[(1 + x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n, \quad |x| < 1,\]

where

\[\binom{k}{0} = 1, \quad \binom{k}{n} = \frac{k(k-1) \cdots (k-n+1)}{n!} \quad \text{for } n \geq 1.\]

a) Use this to write down the Taylor series for \(\sqrt{1 + x^2}\) at 0. No simplification necessary—leave the binomial coefficients in your answer.

b) Write out the first four terms \((n = 0 \text{ through } n = 3)\) in this series and simplify the coefficients as much as you can.

c) What is the maximum error if the first three terms \((n = 0 \text{ through } n = 2)\) of the series are used to approximate \(\sqrt{1 + x^2}\) in the range \(0 < x < 1/2\)? (Hint: it’s an alternating series.)

2. Repeat problem (1) for the function \(\frac{1}{\sqrt{1 + x}}\).

a) 

b) 

c)
3. Find the value of the series
   a) \( 1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \ldots \)
   b) \( \frac{\pi}{3} - \frac{\pi^3}{3^33!} + \frac{\pi^5}{3^55!} - \frac{\pi^7}{3^77!} + \ldots \)

4. Let \( f(x) = x^3 \cos(x^2) \). Find the value of \( f^{(11)}(0) \). Hint: this is very easy using series.

5. Use series to evaluate the limits:
   \[
   \lim_{{x \to 0}} \frac{\sin(x^2) - x^2 \cos x}{x^4}.
   \]