INSTRUCTIONS:

- DO NOT OPEN YOUR EXAM UNTIL YOU ARE TOLD TO DO SO

- The exam is worth 150 points. There are 12 questions. PLEASE ATTEMPT TEN QUESTIONS. MARKS WILL BE AWARDED TO THE 10 QUESTIONS SELECTED.

- This is a closed book exam. No calculators are allowed

- No cell phones are allowed. Please TURN OFF ALL ELECTRONIC DEVICES. Any usage of such will be considered cheating.

- If you have a question, raise your hand and a proctor will address your question.

- Write all solutions clearly and eligibly

SCORES:

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TOTAL______________________________
1. The Region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about the x-axis. Find the volume of the resulting solid. [15 marks]
2. (a) Find the average value of the function \( g(x) = x^2 \sqrt{1 + x^3} \), on the interval [0,2]. [15 marks]
3. Find the area of the region bounded by the curves 
\[ y = \sin x, y = \cos x, x = 0 \text{ and } x = \frac{\pi}{2}. \]

[15 marks]
4. A particle moves in a straight line and has an acceleration given by \( a(t) = 6t + 4 \). Its initial velocity is \( v(0) = -6 \text{ cm/s} \) and its initial displacement is \( s(0) = 9 \text{ cm} \). Find its position function \( s(t) \).
   [15 marks]

5. Find \( f \) if \( f'(x) = e^x + 20(1 + x)^{-1} \) and \( f(0) = -2 \)  
   [15 marks]
6. The Urbana Park District is planning to build a picnic park for motorists along a major highway. The park is to be rectangular with an area of 5,000 square yards and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing required for this job? How long and wide should the park be for the fencing to be minimized?  [15 marks]
7. Find the equation of the tangent line to the curve \( x^2y^3 - 2xy = 6x + y + 1 \) at \((0,-1)\) \hspace{1cm} [15 marks]
8. Find the equation of the normal to the curve $y = (x + \ln{x})^3$ where $x = 1$
[15 marks]
9. State True or false for the following [15 marks]

(i) the graph of an even function is symmetric about the x-axis

(ii) The graph of an odd function is symmetric about the origin

(iii) A vertical line intersects the graph of a function at most once

(iv) If \( x_1 < x_2 \) and \( f \) is a decreasing function, then \( f(x_1) > f(x_2) \).

(v) If \( x > 0 \), then \( (\ln x)^6 = 6\ln x \)

(vi) If \( f \) and \( g \) are differentiable then \( \frac{d}{dx} [f(g(x))] = f'(g(x))g'(x) \)

(vii) If \( f \) and \( g \) are differentiable then \( \frac{d}{dx} \sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}} \)

(viii) If \( f \) and \( g \) are differentiable then \( \frac{d}{dx} f(\sqrt{x}) = \frac{f'(\sqrt{x})}{2\sqrt{x}} \)

(ix) if the distance travelled by an object is \( s(t) = 15t^3 - 4 \) then the acceleration is \( a(t) = 90t \)

(x) \( \frac{d}{dx} (10^x) = x10^{x-1} \)

(xi) \( \frac{d}{dx} (\ln 10) = \frac{1}{10} \)

(xii) If \( f \) is differentiable at \( a \) then \( f \) is continuous at \( a \)

(xiii) \( \frac{d}{dx} (sechx) = tanh^2 x \)

(xiv) If \( f \) is continuous on \([a, b]\) then \( \int_{a}^{b} 5f(x)dx = -5 \int_{a}^{b} f(x)dx \)

(xv) \( \lim_{x \to 0} \frac{x}{e^x} = 1 \)
10. Using logarithmic differentiation. Find the derivative of \( y = x^{\sqrt{2x}} \) [15 marks]
11. Consider the function \( f(x) = e^x - 12x + ln2 \)

(a). The function \( f(x) \) has only one critical point. Find the \( x \)-coordinate of this lone critical point. [15 marks]

(b). Use the second derivative test to classify the critical point found in part (a) as a local maximum or local minimum. If the second derivative test fails explain why.

[ 10 marks]
12. Use the following information to make an accurate sketch of $f(x)$:

$$f(-4) = -2, f(0) = 3, \quad f(3) = 0$$

$$\lim_{x \to -2^-} f(x) = -\infty, \quad \lim_{x \to -2^+} f(x) = -\infty$$

$$\lim_{x \to -\infty} f(x) = \infty, \quad \lim_{x \to \infty} f(x) = 8$$

$f'(x)$ is positive on: $(-2, 0)$ and $(3, \infty)$

$f'(x)$ is negative on: $(-\infty, -2)$ and $(0, 3)$

$f''(x)$ is positive on: $(-\infty, -4)$

$f''(x)$ is negative on: $(-4, -2), (-2, 3), and (3, \infty)$ [15 marks]
This was a final exam given in summer 2011 by A. Hunte. This is given as a practice exam in fall 2011 to sections AD1, DD8 by A. Eid.
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