Announcements

\[ \{ \text{functions} \} \xrightarrow{\nabla} \{ \text{vector fields} \} \xrightarrow{\text{curl}} \{ \text{vector fields} \} \xrightarrow{\text{div}} \{ \text{functions} \} \]

\[ f(p) \quad \text{FTLI} \quad \int \int \int \quad \text{Stokes' Thm.} \quad \iiint \quad \text{Divergence} \quad \iint \int \quad \text{fundamental theorem of line integrals} \]

\[ \{ \text{points} \} \xrightarrow{\Theta} \{ \text{curves} \} \xrightarrow{\Theta} \{ \text{surfaces} \} \xrightarrow{\Theta} \{ \text{solids} \} \]

1) \( \nabla \cdot \int_C \mathbf{F} \cdot d\mathbf{r} \)
2) \( \Theta: \quad f(q) - f(p) \)

Stokes' Theorem:

1) \( \text{curl} : \quad \iiint \text{curl} \mathbf{F} \cdot d\mathbf{S} \)
2) \( \Theta : \quad \int_S \mathbf{F} \cdot d\mathbf{a} \)

Divergence Theorem:

1) \( \text{div} : \quad \iiint_E \text{div} \mathbf{F} \ dV \)
2) \( \Theta : \quad \int_{\partial E} \mathbf{F} \cdot d\mathbf{S} \)

1) Any two consecutive arrows on the top row give \( 0 \)
2) Any two consecutive arrows on the bottom row give the empty set.
3) Can integrate/evaluate any column to get a number
4) Each square is a theorem
Approaches to solving problems.

1. If $\mathbf{F} = \nabla g$, use fundamental theorem of line integrals.

2. Solve directly if possible.

3. Are you in $\mathbb{R}^2$ or $\mathbb{R}^3$?

   - $\mathbb{R}^2$: Use Green's theorem.
     - Choose $D \subset \mathbb{R}^2$ so that $\mathbf{F}$ is nice on $D$.
     - (a) Ideal: $\partial D = \pm C$
     - (b) Okay: $\partial D = \pm C \cup C'$, where $\int_C \mathbf{F} \cdot d\mathbf{r}$ is "easy".

   - $\mathbb{R}^3$: Use Stokes' theorem.
     - Find oriented surface so $\mathbf{F}$ is nice on $S$.
     - (a) Ideal: $\partial S = C$
     - (b) Okay: $\partial S = C \cup C'$ and $\int_C \mathbf{F} \cdot d\mathbf{r}$ is "easy".

4. Approximate: Use $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_{C'} (\text{length of } C) \text{(average value of } \mathbf{F} \cdot d\mathbf{r})$.

   - Use Riemann sum.

5. If $\mathbf{F} = \nabla \Phi$ is given, use Stokes' theorem.

6. Solve directly if possible.

7. Use the divergence theorem.
   - Find a solid $E$ so that $\mathbf{F}$ is nice on $E$.
     - (a) Ideal: $\partial E = \pm S$
     - (b) Okay: $\partial E = \pm S \cup S'$ and $\int_S \mathbf{F} \cdot d\mathbf{S}$ is "easy".

8. Approximate.

What's the best way?

1. Follow instructions.

2. Do the easiest problem:
   - Squiggles and blobs are hard.
   - Polygons and boxes are hard.
   - Complicated near fields are hard.
   - Already parameterized surfaces.

3. Do steps in order.

4. "Approximate" and "positive/negatives" are easier.

5. Don't be afraid to waste a little time, but don't waste too much time.