Fix $r > 0$ and let $C_r = \{x^2 + y^2 = r^2\}$, oriented counter-clockwise. Let $\mathbf{F}(x, y) = \langle -y, x \rangle$ be a vector field on $\mathbb{R}^2$.

Choose a parametrization of $C_r$, and use it to calculate the integral

$$\int_{C_r} x \, dy - y \, dx = \int_{C_r} \mathbf{F} \cdot d\mathbf{r}.$$ 

(a) $-2\pi r^2$
(b) 0
(c) $-\pi r^2$
(d) $2\pi r^2$
(e) I can't remember how to do this.
Midterm 3 is next Tuesday, April 16, 7–8:15pm.

- The rooms are *not* the same as last time. Make sure you check the exam webpage carefully.
- The exam process is not quite the same as last time, either. In particular, there will be multiple versions of the exam, and the way you will be assigned seats is different. Pay attention to your TA’s instructions.
- If you need to take the conflict exam, you must fill out the conflict exam request form by tomorrow.
Recall some important theorems

A path is a piecewise smooth curve.

**Fundamental Theorem of Calculus**

\[ \int_a^b f'(x) \, dx = f(b) - f(a). \]

**Fundamental Theorem of Line Integrals**

Let \( C \) be a path from \( A \) to \( B \).

\[ \int_C \nabla f \cdot dr = f(B) - f(A). \]

Note: we have derivatives on the left, and boundary terms appearing on the right.
Assumptions for today

- \( \mathbf{F} = \langle P, Q \rangle \) has continuous first order partial derivatives on an open set \( D \subset \mathbb{R}^2 \).

- \( B \subset D \) is “nice”:
  - We can integrate over \( B \).
  - The boundary \( \partial B \) is one or more simple closed paths.

- We orient \( \partial B \) so that \( B \) is always on the left.
Fix $r > 0$, and let $B_r = \{x^2 + y^2 \leq r\}$. Use Green’s Theorem (in particular, part (C) of the last theorem) to find the area of $B_r$.

(a) Got it!
(b) I don’t see what to do yet.
Practice applying Green’s theorem

Let $\mathbf{F} = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$. Recall that $P_y = Q_x$. Which of the following arguments is correct?

(a) On $C_r$, $\langle P, Q \rangle = \langle \frac{-y}{r^2}, \frac{x}{r^2} \rangle$, so

$$\int_{C_r} \mathbf{F} \cdot d\mathbf{r} = \frac{1}{r^2} \int_{C_r} x \; dy - y \; dx = \frac{2\pi r^2}{r^2} = 2\pi.$$

(b) By Green’s Theorem,

$$\int_{C_r} \mathbf{F} \cdot d\mathbf{r} = \int \int_{B_r} (Q_x - P_y) dA = \int \int_{B_r} 0 dA = 0.$$
Using Green’s Theorem

Let $\mathbf{F} = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$ as before, and let $C'$ be a simple closed curve in $\mathbb{R}^2$ enclosing the origin $(0,0)$. What is $\int_{C'} \mathbf{F} \cdot d\mathbf{r}$?

(a) There is not enough information to answer the question.
(b) 0.
(c) $2\pi$.
(d) $-2\pi$.
(e) I don’t know.