Last time: integrating in polar coordinates

The equation \( r = \cos 2\theta \) traces out a “four-leafed rose” as \( \theta \) varies. In particular, a leaf or petal of the rose is the region enclosed one loop of the curve, given by

\[
D = \{(r, \theta) \mid \frac{-\pi}{4} \leq \theta \leq \frac{\pi}{4}, 0 \leq r \leq \cos 2\theta\}.
\]

Sketch the curve. Which formula can be used to calculate the area of this leaf?

(a) \( \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{0}^{\cos 2\theta} rdrd\theta \)

(b) \( 2 \int_{0}^{\frac{\pi}{4}} \int_{0}^{\cos 2\theta} rdrd\theta \)

(c) \( \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{0}^{\cos 2\theta} rdrd\theta \)

(d) \( \int_{0}^{\cos 2\theta} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} rd\theta dr \)

(e) I don’t know.
Practice with triple integrals

We wrote

\[ E = \{(x, y, z) \mid (x, z) \in D_2, x^2 + z^2 \leq y \leq 4\}, \]

where \( D_2 = \{(x, z) \mid x^2 + z^2 \leq 4\}. \) so

\[
V(E) = \iiint_E dV = \int \int \int_{D_2} \int_x^4 dy \ dA = \int \int \int_{D_2} [y]^4_{x^2+z^2} \ dA = \int \int \int_{D_2} 4 - x^2 - z^2 dA.
\]
Practice with triple integrals

So we need to calculate $\int\int_{D_2} 4 - x^2 - z^2 dA$. We could write $D_2$ as a region of type I or II, but it is easier to use polar coordinates (in the $xz$-plane):

$$x = r \cos \theta, \quad z = r \sin \theta;$$

$$\int\int_{D_2} 4 - x^2 - z^2 dA = \int_0^{2\pi} \int_0^2 (4 - r^2)r \ drd\theta$$

$$\int_0^{2\pi} \left[ 2r^2 - \frac{1}{4}r^4 \right]_0^2 d\theta$$

$$\int_0^{2\pi} 4d\theta = 8\pi.$$
Practice with triple integrals

$E$ is the region bounded by the planes $x = 2, y = 2, z = 0$ and $x + y - 2z = 2$, and $D$ is its triangular shadow in the $xy$-plane.

Fill in the blanks, and discuss with your neighbour:

$$D = \{(x, y) \mid \underline{?} \leq x \leq \underline{?}, \underline{?} \leq y \leq \underline{??}\}.$$  

(a) We’re working on it.
(b) We’re stuck.
(c) We have answers, but they’re different and we don’t know who is right.
(d) We have the same answer.
Practice with triple integrals

Now find \( u_1(x, y) \) and \( u_2(x, y) \) such that

\[
E = \{(x, y, z) \mid (x, y) \in D, \quad u_1(x, y) \leq z \leq u_2(x, y)\}.
\]

(a) We’re working on it.
(b) We’re stuck.
(c) We have answers, but they’re different and we don’t know who is right.
(d) We have the same answer.