Question: Let $Q$ be an $n$-gon, not necessarily $g$-convex, and suppose you add a triangle $T$ to $Q$ by identifying a side of $T$ with a side of $Q$.

Depending on whether the two other sides of $T$ form $180^\circ$ angles with the sides of $Q$, one obtains an $n+1$-gon, an $n$-gon, or a $(n-1)$-gon $P$.

Show that in each case, the defect of $P$ is $\text{defect} \ (T) + \text{defect} \ (Q)$.

Now suppose you add a triangle $T$ to $Q$ such that two of $T$'s sides are identified with sides of $Q$.

The resulting $n$-gon $P$ has $n-1$, $n-2$ or $n-3$ sides.

Show that in each case, the defect of $P$ is $\text{defect} \ (Q) + \text{defect} \ (T)$.

Example: P has $n-2$ sides.

\[
\text{Defect} \ (P) = 180(n-2) - \text{(angle sum of } P) \\
= 180(n-4) - \left[ \text{(angle sum of } Q \right) + \text{(angle sum of } T) - 360 - 180 \right] \\
= 180(n-1) - \left[ \text{angle sum of } Q \right] + \text{angle sum of } T] \\
= (180(n-2) - \text{angle sum of } Q) + (180 - \text{angle sum of } T) \\
= \text{defect } Q + \text{defect } T.
\]
Nas given any polygon $P$ which has been divided into triangles $T_1, \ldots, T_n$. We want to show $\text{defect}(P) = \sum_{i=1}^{n} \text{defect}(T_i)$.

WLOG assume that all vertices in the triangulation are common vertices (if not, add more lines).

Then we'll show

\[
\text{defect}(T_i) = \text{defect}(S_i) + \text{defect}(S_i)
\]

and $\text{defect}(P) = \text{defect}(S_1) + \text{defect}(S_2) + \text{defect}(T_1) + \text{defect}(T_2)$,

which implies $\text{defect}(P) = \sum_{i=1}^{n} \text{defect}(T_i)$.

A triangle in such a triangulation shares exactly 1, 2 or 3 sides with other triangles (i.e., has 0, 1, or 2 sides exposed).

Now by induction on the number of triangles in the triangulation, show that $\text{defect}(P) = \sum_{i=1}^{n} \text{defect}(T_i)$.

(Use results on the previous page)