Exercise 1.

a. [8 points] Suppose you have a triangle \( \triangle ABC \) in which \( AB \neq BC \). (Without loss of generality, you can assume that \( AB > BC \).) Prove that the angles opposite these sides are also not congruent, and in fact the angle opposite the longer side \( AB \) is larger than the angle opposite the shorter side \( BC \). (Hint: extend the shorter side to make an isosceles triangle. You may wish to use the Exterior Angle Theorem.)

Definition 1. Let \( \ell \) be a line and \( P \) a point not on \( \ell \). Drop the perpendicular line from \( P \) to \( \ell \), and call the intersection point \( Q \). We say that the distance from \( P \) to \( \ell \) is the length of the segment \( PQ \).

b. [6 points] Prove that if \( Q' \) is any other point on \( \ell \), \( PQ' > PQ \).

c. [8 points] Prove that Playfair’s Postulate (or the Parallel Postulate, or other results we’ve already proved using them) implies the following:

   Let \( \ell \) and \( m \) be two parallel lines, and let \( A, B \) be any two points on \( m \). Then the distance from \( A \) to \( \ell \) is equal to the distance from \( B \) to \( \ell \). (We say that \( \ell \) and \( m \) are equidistant.)

d. [8 points] Prove the converse: if any two parallel lines are equidistant, Playfair’s Postulate must hold. (Try a proof by contradiction: suppose you have two lines \( m \) and \( n \) parallel to \( \ell \) and both passing through some point \( P \). Choose a point \( X \) on \( \ell \) and think about its distance from \( m \) and \( n \).)

Exercise 2. The point of this exercise is to reinforce the material you learned in this week’s project.

a. [2 points] Let \( c \) be a circle with centre \( O \) and radius \( r \). Let \( P \) be any point. Define the power of \( P \) with respect to the circle \( C \).

b. [3 points] Prove that the power of \( P \) is positive when \( P \) is outside the circle, negative when \( P \) is inside the circle, and zero when \( P \) is on the boundary of the circle.

Exercise 3. [10 points] Prove the SSS similarity property: if you have two triangles such that the sides are all in proportion, the triangles are similar. (You may use the similarity properties we have already proved in lecture: AAA similarity and/or SAS similarity.)

Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.