Exercise 1.
a. Let \( \ell \) be a line and \( c \) a circle. How many points can there be in the intersection of \( \ell \) and \( c \)? (You will get several cases. To justify your answer, draw a picture showing how each of the cases occurs, and then prove that there are no other cases.)
b. Let \( c_1 \) and \( c_2 \) be two circles. How many points can there be in the intersection of \( c_1 \) and \( c_2 \)? Justify your answer as above.
c. What does it mean for a point \( P \) to be inside the circle \( c \)? Give a definition. The answer is not in the book; you need to formulate a mathematical definition that accurately describes the picture you can draw of a point inside a circle.
d. Prove (following your definition from above) that if \( P \) is a point inside of \( c \) and \( Q \) is a point outside of \( c \), then the segment \( \overline{PQ} \) intersects \( c \).

Definition 1. A line \( \ell \) is said to be tangent to a circle \( c \) if \( \ell \) intersects \( c \) at exactly one point \( T \). (This point is called the point of tangency.)

Exercise 2. Prove the following (using the book for guidance).
a. Given a circle \( c \) with centre \( O \) and radius \( \overrightarrow{OT} \), if \( \ell \) is tangent to \( c \) at \( T \), then \( \ell \) is perpendicular to \( \overrightarrow{OT} \) at \( T \).
b. Conversely, given the same circle, if \( \ell \) is perpendicular to \( \overrightarrow{OT} \) at \( T \), then \( \ell \) is tangent to \( c \) at \( T \).

Definition 2. Two distinct circles are said to be mutually tangent at a point \( T \) if the same line through \( T \) is tangent to both circles at \( T \).

Exercise 3. Let \( c, c' \) be two circles which are mutually tangent at the point \( T \). Let \( \ell \) be the line which is tangent to both circles at \( T \), and let \( O, O' \) be the centre points of the two circles.
a. Show that the line \( \overrightarrow{OO'} \) passes through \( T \).
b. Now show that the circles \( c \) and \( c' \) intersect in only one point. To do this, assume (towards a contradiction) that they intersect at another point \( P \). Construct two isosceles triangles, and then use your result from a. to derive a contradiction.

You do not need to hand your work in, but you are expected to complete it. If you get stuck or are unsure about your answers, come to office hours. This material is examinable and will not be covered in ordinary lecture format, so you must make sure that you understand it as it is presented here.