MATH 402 Review worksheet

Topics: material covered in class (through lectures, worksheets, projects, or homework) by the exam date. This corresponds to sections 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 2.1, 2.2, 2.5, 2.6, 2.7, 3.1, 3.2, 3.4, 7.1, 7.2.

1. All problems from homework, worksheet, and projects, as well as other problems from the book.

2. Definitions and statements:

   (a) Define parallel lines, perpendicular lines.
   (b) State Euclid’s 5th postulate and Playfair’s postulate.
   (c) Define congruent triangles, similar triangles.
   (d) State Pasch’s axiom.
   (e) Know the undefined terms of Hilbert’s axiom system: point, line, congruence (of line segments, and of angles), betweenness, and incidence (the property of a point being on a line).
   (f) State the supplementary angle theorem, the vertical angle theorem, and the exterior angle theorem.
   (g) State SAS/SSS/ASA congruence rules, and SSS/AAA similarity rules.
   (h) Basic geometric constructions: an equilateral triangle given a base; the perpendicular bisector of a segment; the bisector of an angle.
   (i) Give careful definitions (based on Hilbert’s axioms): equilateral triangle; two points being on the same side of a line; two points being on opposite sides of the line; a point being on the interior of an angle.
   (j) Define neutral geometry.
   (k) Define a chord, diameter, major/minor arc, central angle, inscribed angle, tangent line to a circle.
   (l) Know that an inscribed angle has half the measure of the corresponding central angle.
   (m) Know that a line is tangent to a circle if and only if it is perpendicular to the radius at the point of tangency.
   (n) Define orthogonal and mutually tangent circles.
   (o) Define the inverse of a point with respect to a circle.
   (p) Give the equation of a line and a circle in analytic geometry.
   (q) State the hyperbolic parallel postulate.
   (r) Define points, lines, and angles in the Poincaré module. Define the Poincaré distance.
   (s) Define points and lines in the Klein model. Define the Klein distance. State what it means for two Klein lines to be perpendicular. Define limiting parallels and the angle of parallelism.
3. Proofs:

(a) Prove the vertical angle theorem.
(b) Prove the exterior angle theorem.
(c) Given Hilbert’s axioms, prove SSS and ASA.
(d) Prove the Isosceles Triangle Theorem.
(e) Consider the axiomatic system defined by the following. The undefined terms are points, and a line is defined as a set of points. The axioms are:
   i. There are exactly four points.
   ii. There are exactly four lines.
   iii. Given any two different points, there is at least one line that contains them.

   • I claim that this system is consistent. Give a model.
   • Show that each of these three axioms is independent from the others.
   • Is the system complete? Why or why not?
   • True or false: Every line contains at most two points. Justify your answer.
   • From the given system \( S \) form a new one \( T \) which is ‘dual’: that is, points in \( T \) are lines in \( S \), and lines in \( T \) are points in \( S \). Does \( T \) satisfy the same axioms as \( S \)? If the answer is yes, prove your claim by proving that each axiom holds for \( T \). If the answer is no, prove that one of the axioms for \( S \) (which one?) does not hold.

4. The definition of group.

True/false questions

Decide whether each of the following statements is true or false. Make sure you know a proof or a counterexample in each case.

1. In neutral geometry, given any two parallel lines there is at least one line which is perpendicular to both of them.
2. Let \( c \) be a (Euclidean) circle with origin \( O \). There is no circle \( c' \) which goes through \( O \) and which is orthogonal to \( c \).
3. In Euclidean geometry, if \( \ell_1 \) and \( \ell_2 \) are two unequal parallel lines, and \( m \) is another line which intersects \( \ell_1 \) (but is not equal to \( \ell_1 \)), then \( m \) must intersect \( \ell_2 \).
4. In hyperbolic geometry, if \( \ell_1 \) and \( \ell_2 \) are two unequal parallel lines, and \( m \) is another line which intersects \( \ell_1 \) (but is not equal to \( \ell_1 \)), then \( m \) must intersect \( \ell_2 \).

There will be more true/false questions in class on Wednesday to practice what you’ve reviewed.