Exercise 1. a. [3 pts] Let $T = r_{\ell_2} \circ r_{\ell_1}$ be a translation, with displacement vector $v$. Prove that the inverse of $T$ is also a translation, given by $r_{\ell_1} \circ r_{\ell_2}$ and having displacement vector $-v$.

b. [3 pts] Let $T_1$ and $T_2$ be two translations, with displacement vectors $v_1$ and $v_2$ respectively. Prove that $T_1 \circ T_2$ is again a translation. What is its displacement vector?

c. [3 pts] Show that composition of translations commutes: that is, that $T_1 \circ T_2$ is equal to $T_2 \circ T_1$. Is this true for reflections? Prove or provide a counter-example.

d. [3 pts] Does the set of translations form a group?

Exercise 2. [10 pts] Let $T$ be a translation which is not the identity. Prove that $\ell$ is an invariant line for $T$ if and only if $\ell$ is parallel to the displacement vector $v$ of $T$.

Exercise 3. a. [8 pts] Suppose we are given a coordinate system with origin $O$. Let $\text{Rot}_\phi$ denote rotation about $O$ by angle $\phi$. Let $C = (x, y)$ be a point not equal to $O$, and let $T$ denote the translation with displacement vector $v = (x, y)$. Prove that $T \circ \text{Rot}_\phi \circ T^{-1}$ is rotation about $C$ by angle $\phi$.

b. [8 pts] Given a coordinate system with origin $O$, let $\ell$ be a line which does not pass through $O$. Using translations, rotations, and reflection across the $x$-axis, give an expression for reflection $r_\ell$ across $\ell$.

Exercise 4. a. [2 pts] Let $\text{Rot}_\phi$ be rotation about a point $O$ by angle $\phi$. Use reflections to prove that the inverse of $\text{Rot}_\phi$ is rotation about $O$ by angle $-\phi$.

b. [3 pts] Let $\text{Rot}_{\psi}$ be rotation about the same point $O$ by angle $\psi$. Use reflections to prove that $\text{Rot}_\phi \circ \text{Rot}_{\psi}$ is again a rotation about $O$.

c. [4 pts] Let $A$ and $B$ be two different points. Let $R_1$ be rotation about $A$ by $180^\circ$, and let $R_2$ be rotation about $B$ by $180^\circ$. Prove that $R_2 \circ R_1$ is a translation. What is the displacement vector?

d. [3 pts] Let $\mathcal{R}$ denote the set of all rotations. Let $\mathcal{R}_O$ denote the set of all rotations with centre of rotation $O$. Is $\mathcal{R}$ a group? What about $\mathcal{R}_O$?

Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.