** A review of some important calculus topics

Note: Problems marked with an asterisk (*) are Merit problems. All other problems come from the regular Math 241 worksheet, although I may have modified them slightly. You will find the original discussion worksheets and their solutions on the Math 241 course webpage.

1. Chain Rule:
   (a.0) Write on the board what the chain rule states.
   (a) Let $h(t) = \sin(\cos(\tan t))$. Find the derivative with respect to $t$. *What functions are composed to get to $h(t)$?
   (b) Let $s(x) = \sqrt{x}$ where $x(t) = \ln(f(t))$ and $f(t)$ is a differentiable function. Find $\frac{ds}{dt}$.

2. Parameterized curves:
   (a) Describe and sketch the curve given parametrically by
   \[
   \begin{align*}
   x &= 5\sin(3t) \\
   y &= 3\cos(3t)
   \end{align*}
   \quad \text{for} \quad 0 \leq t < \frac{2\pi}{3}.
   
   What happens if we instead allow $t$ to vary between 0 and $2\pi$?
   (b.0) Write the formula for the arc length of a parameterized curve on the board.
   (b) Set up, but **do not evaluate** an integral that calculates the arc length of the curve described in part (a).
   (c) Consider the equation $x^2 + y^2 = 16$. Graph the set of solutions of this equation in $\mathbb{R}^2$ and find two different parameterizations that each traverse the curve once counterclockwise.
   (d) How is the parameterization you came up with in part (c) similar to and different from the one given in part (a)?

3. 1st and 2nd Derivative Tests:
   (a.0) Describe the 1st and 2nd Derivative Tests. How are they similar? How are they different?
   (a) Use the 2nd Derivative Test to classify the critical numbers of the function $f(x) = x^4 - 8x^2 + 10$.
   (b) Use the 1st Derivative Test and find the extrema of $h(s) = s^4 + 4s^3 - 1$.
   (c) Explain why the 2nd Derivative test is unable to classify all the critical numbers of $h(s) = s^4 + 4s^3 - 1$.

4. Consider the function $f(x) = x^2e^{-x}$.
   (a.0) Draw an arbitrary continuous function on the board which goes through the point (2,3). Then, draw the line that best approximates the function you drew at the point $x = 2$. What do we call this line?
(a) Find the best linear approximation to $f$ at $x = 0$.

(b) Compute the second-order Taylor polynomial at $x = 0$.

(c) Explain how the second-order Taylor polynomial at $x = 0$ demonstrates that $f$ must have a local minimum at $x = 0$.

5. Consider the integral $\int_{0}^{\sqrt{3}\pi} 2x \cos(x^2) \, dx$.

(a) Sketch the area in the $xy$-plane that is implicitly defined by this integral.

(b) To evaluate, you will need to perform a substitution. Choose a proper $u = f(x)$ and rewrite the integral in terms of $u$. Sketch the area in the $uv$-plane that is implicitly defined by this integral.

(c) Evaluate the integral $\int_{0}^{\sqrt{3}\pi} 2x \cos(x^2) \, dx$. 