(1) Problem 1.4.3 (b) in [FIS]

(2) Problem 1.4.5 (f) in [FIS]

(3) Problem 1.4.6 in [FIS] (with $F = \mathbb{R}$.)

(4) Problem 1.4.12 in [FIS]

(5) Determine whether each of the following sets is linearly independent.

(a) $S = \{8x^3 + 3x^2 - x - 2, 5x^3 + x^2 + x + 1, 7x^3 + 4x^2 - 3\}$ in $P_3(\mathbb{R})$.

(b) $S = \{\sin^2(x), \cos^2(x), 2 \cos(2x)\}$ in $\mathcal{F}$, where $\mathcal{F}$ is the vector space of all functions $f : \mathbb{R} \to \mathbb{R}$.

(6) Suppose $S = \{v_1, \ldots, v_n\}$ is a linearly independent subset of a vector space $V$ and $w \in V$.

(i) Prove that if $\{v_1 + w, \ldots, v_n + w\}$ is linearly dependent, then $w \in \text{span}(S)$.

(ii) Is the converse of (i) true? If it is true, give a proof. If it is false, give a counterexample.

* [FIS] = Friedberg, Insel, Spence’s Linear Algebra text (4th edition)