1. Let $R$ be the region of integration pictured below right. Evaluate $\iint_R 6y \, dA$. (4 points)

\[
\int_0^1 \int_{-y}^{y+1} 6y \, dx \, dy = \int_0^1 6y(2y+1) \, dy = \int_0^1 12y^2 + 6y \, dy = 4y^3 + 3y^2 \bigg|_{y=0}^{y=1} = 7
\]

$\iint_R 6y \, dA = 7$

2. Set up, but DO NOT EVALUATE, a triple integral that computes the volume of the tetrahedron shown at right. (5 points)

Two common correct answers:

\[\begin{align*}
&\text{(a)} \quad \iiint 1 \, dz \, dy \, dx \\
&\text{(b)} \quad \iiint 1 \, dz \, dx \, dy
\end{align*}\]

3. Set up, but DO NOT EVALUATE, a triple integral that computes the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and above the cone $z = \sqrt{x^2 + y^2}$. (5 points)

Cross section:

3D:

Using spherical coordinates:

\[
\int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sqrt{2}} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = 1 \, dV
\]
4. Consider the triple integral \( \int_0^{1/2} \int_0^{1-y} \int_0^{1-x^2-y^2} f(x, y, z) \, dz \, dx \, dy \). Mark the corresponding region of integration below. (3 points)

5. Find a transformation \( T: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) which takes the unit circle to the ellipse given by \((x - 3)^2 + \frac{y^2}{4} = 1\) as shown. (3 points)
6. Let \( R \) be the region in the \( xy \)-plane depicted below right. Let \( T(u,v) = (2u + v, u - v) \).

(a) Find a rectangle \( S \) in the \( uv \)-plane whose image under \( T \) (that is, the collection of points \( T(u,v) \) for all choices of \( (u,v) \) in \( S \)) is exactly \( R \). \( \text{(3 points)} \)

\[
\begin{align*}
T(1,0) &= (2,1) \\
T(0,1) &= (1,-1)
\end{align*}
\]

So \( 0 \leq u \leq 1 \) and to get image down to \((2,-2)\) want:

**ANSWER:** \( S = \{(u,v) \mid 0 \leq u \leq 1, \; 0 \leq v \leq 2\} \).

(b) Set up, but DO NOT EVALUATE, the integral \( \iint_R \cos(x) \, dA \) as an integral in the \((u,v)\)-coordinates. If you can’t do part (a), leave the limits of integration blank. \( \text{(5 points)} \)

\[
\begin{align*}
\iint_R \cos(x) \, dA &= \iint_S (2u + v) \, |\det J| \, du \, dv \\
J &= \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \\
\det J &= -2 - 1 \\
&= -3
\end{align*}
\]

\[
\iint_R \cos(x) \, dA = \int_0^2 \int_0^1 3 \cos(2u + v) \, du \, dv
\]

7. Let \( F(x,y) = (x^2, x^2 \cos(y)) \). Then \( \iint_R \left[ \frac{\partial}{\partial x} (x^2 \cos(y)) - \frac{\partial}{\partial y} (x^2) \right] \, dA = 0 \) where \( R \) is the region shown below.

Compute \( \int_C F \cdot dr \), where \( C \) is the pictured curve that goes from \((-3,0)\) to \((3,0)\) via \((0,2)\). \( \text{(3 points)} \)

By Green, we have \( \int_C \vec{F} \cdot d\vec{r} = \iint_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA = 0 \)

Thus \( \int_C F \cdot dr = \int_{C'} F \cdot dr \)

\[
\begin{align*}
\int_C x^2 \, dx + \int_{C'} x^2 \cos(y) \, dy &= \left[ \frac{x^3}{3} \right]_{-3}^3 \\
&= 18
\end{align*}
\]

\( C' \) = bottom of \( R \).
8. Consider the region $D$ in the plane bounded by the curve $C$ as shown at right. For each part, circle the best answer. (1 point each)

(a) For $\mathbf{F}(x, y) = \langle x + 1, y^2 \rangle$, the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is

- negative
- zero
- positive

(b) The integral $\int_C (-y \, dx + 2 \, dy)$ is

- negative
- zero
- positive

(c) The integral $\iint_D (y - x) \, dA$ is

- negative
- zero
- positive

9. For each surface $S$ below, give a parameterization $\mathbf{r}: D \to S$. Be sure to explicitly specify the domain $D$ and call your parameters $u$ and $v$.

(a) The rectangle in $\mathbb{R}^3$ with vertices $(1,0,0), (0,1,0), (1,0,2), (0,1,2)$. (3 points)

Can use coordinates $x$ and $z$ as parameters, call them $u$ and $v$. Since this rect. is contained in the line $x+y=1$, get

$y = 1-x = 1-u$

$D = \{ 0 \leq u \leq 1 \text{ and } 0 \leq v \leq 2 \}$

$\mathbf{r}(u,v) = \langle u, 1-u, v \rangle$

(b) The portion of cone $y = \sqrt{x^2 + z^2}$ for $0 \leq y \leq 1$ which is shown at right. (4 points)

Can use $v = y$ and the angle $u$ shown as the parameters. Also, the radius of the circle with $y$ fixed is just $y$, and hence:

$D = \{ 0 \leq u \leq 2\pi \text{ and } 0 \leq v \leq 1 \}$

$\mathbf{r}(u,v) = \langle v \cos u, v, v \sin u \rangle$

10. Consider the surface $S$ parameterized by $\mathbf{r}(u, v) = (v^2, u, v)$ for $0 \leq u \leq 1$ and $0 \leq v \leq 1$.

(a) Mark the correct picture of $S$ below. (2 points)

(b) Evaluate the integral $\iint_S z \, dA$. (6 points)

$$
\iint_S z \, dA = \int_0^1 \int_0^1 \sqrt{1 + 4v^2} \, du \, dv = \int_0^1 \int_0^1 \sqrt{1 + 4v^2} \, du \, dv
$$

$$
= \int_0^1 \sqrt{1 + 4v^2} \, dv = \int_1^8 \frac{1}{8} \, dw
$$

$$
dw = 8v \, dv
$$

$$
= \frac{1}{12} \int_0^1 w^{3/2} \Big|_{w=1}^{w=5} = \frac{1}{12} (5^{3/2} - 1)
$$

$$
\iint_S z \, dA = \frac{1}{12} (5^{3/2} - 1)
$$

11. Consider the solid described as follows using cylindrical coordinates: $E$ is the region inside the paraboloid $z = 1 - r^2$ and where $0 \leq \theta \leq \pi$ and $z \geq 0$. Choose one double integral and one triple integral below that compute the volume of $E$. (1 point each)

- $\int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \sqrt{1-z-x^2} \, dy \, dz$
- $\int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \sqrt{1-z-x^2} \, dx \, dz$
- $\int_0^1 \int_0^{\sqrt{1-z}} 1 \, dz \, dy$
- $\int_0^1 \int_0^{\sqrt{1-z}} 2\sqrt{1-z-y^2} \, dy \, dz$
- $\int_0^1 \int_{\sqrt{1-z}}^{1-z} 1 \, dy \, dz$
- $\int_0^1 \int_{\sqrt{1-z}}^{1-z} 2\sqrt{1-z-y^2} \, dy \, dz$
- $\int_0^1 \int_{\sqrt{1-z}}^{1-z} 2\sqrt{1-z-y^2} \, dy \, dz$
- $\int_0^1 \int_{\sqrt{1-z}}^{1-z} 2\sqrt{1-z-y^2} \, dy \, dz$