1. (a) Consider the points $A = (0, 1, 2)$, $B = (2, 2, 3)$, and $C = (-1, 3, 4)$. Compute the vectors $\vec{v} = \overrightarrow{AB}$ and $\vec{w} = \overrightarrow{AC}$. (2 points)

\[
\vec{v} = \begin{bmatrix} , & , & \end{bmatrix} \\
\vec{w} = \begin{bmatrix} , & , & \end{bmatrix}
\]

(b) Consider the points $D = (0, 2, 1)$, $E = (1, 4, 1)$, and $F = (3, 8, 2)$ and the vectors $\vec{a} = \langle 1, 2, 0 \rangle$, $\vec{b} = \langle 3, 6, 1 \rangle$, and $\vec{c} = \langle 2, 4, 1 \rangle$ as shown at right. Find a normal vector $\mathbf{n}$ to the plane containing the points $D, E,$ and $F$. (3 points)

\[
\mathbf{n} = \begin{bmatrix} , & , & \end{bmatrix}
\]

(c) Let $P = (2, -1, 1)$ and $\mathbf{u} = \langle 3, 2, 4 \rangle$. Find a linear equation for the plane that contains $P$ and has normal vector $\mathbf{u}$. (2 points)

Equation: \[
x + \quad y + \quad z =
\]

(d) For two vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^3$, which of the following does $|\mathbf{v} \times \mathbf{w}|$ measure? Circle your answer. (1 point)

- The length of $\mathbf{v} - \mathbf{w}$.
- The area of the parallelogram determined by $\mathbf{v}$ and $\mathbf{w}$.
- The volume of the parallelepiped determined by $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{v} \times \mathbf{w}$.  

2. (a) Find the midpoint $M$ of the straight-line segment between the points $P = (1,3,-2)$ and $Q = (3,-3,4)$. (2 points)

$$M = (\quad, \quad, \quad)$$

(b) Let $L$ be the line parametrized by $r(t) = \mathbf{a} + t\mathbf{b}$ for $\mathbf{a} = \langle 1,0,2 \rangle$ and $\mathbf{b} = \langle 2,-1,1 \rangle$. Find the point $Q$ of intersection of the line $L$ with the plane whose equation is $3x - 2y + z = 14$. (3 points)

$$Q = (\quad, \quad, \quad)$$

3. Consider the function $f(x,y) = \frac{xy - x^2y}{x^2 + y^2}$ for $(x,y) \neq (0,0)$. Evaluate $\lim_{(x,y) \to (0,0)} f(x,y)$ or explain why it does not exist. (5 points)
4. For each function

(a) \(-x^2 - y^2\)  
(b) \(\cos(x + y)\)  
(c) \(y^2 e^{-x^2}\)

label its graph and its level set diagram from among the options below. Here each level set diagram consists of level sets \(\{ f(x, y) = c_i \} \) drawn for evenly spaced \(c_i\). (9 points)
5. (a) Compute the partial derivatives of \( f(x, y) = x^3 + 2xy + y \). (2 points)

\[
\frac{\partial f}{\partial x} = \]

\[
\frac{\partial f}{\partial y} = \]

(b) Consider the graph of \( f: \mathbb{R}^2 \to \mathbb{R} \) shown at right. If \( P \) is the point \((a, b, f(a, b))\), find the sign of each of the quantities below. Circle your answers. (1 point each)

\[
f_x(a, b): \quad \text{positive} \quad \text{negative} \quad 0
\]

\[
f_y(a, b): \quad \text{positive} \quad \text{negative} \quad 0
\]

\[
f_{xx}(a, b): \quad \text{positive} \quad \text{negative} \quad 0
\]

(c) Use the contour plot of \( f(x, y) \) shown at right to estimate \( f_x(2, 1) \) and \( f_y(2, 1) \). For each, circle the number below that is closest to your estimate. (4 points)

\[
f_x(2, 1): \quad -3 \quad -2.5 \quad -2 \quad -1.5 \quad -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3
\]

\[
f_y(2, 1): \quad -3 \quad -2.5 \quad -2 \quad -1.5 \quad -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3
\]
6. Suppose a function $f: \mathbb{R}^2 \to \mathbb{R}$ has $f(1,2) = 5$, $\frac{\partial f}{\partial x}(1,2) = -2$, and $\frac{\partial f}{\partial y}(1,2) = 3$. Use linear approximation to estimate $f(1.1, 1.9)$. (3 points)

\[ f(1.1, 1.9) \approx \]

7. Suppose $f(x, y): \mathbb{R}^2 \to \mathbb{R}$, $x(s, t): \mathbb{R}^2 \to \mathbb{R}$, and $y(s, t): \mathbb{R}^2 \to \mathbb{R}$ are functions. Let $F(s, t) = f(x(s, t), y(s, t))$ be their composition.

(a) Write the formula for $\frac{\partial F}{\partial s}$ using the Chain Rule. (1 point)

\[ \frac{\partial F}{\partial s} = \]

(b) Suppose $x(s, t) = 2s + t$ and $y(s, t) = s^2 t - 1$ and that $f(x, y)$ has the table of values and partial derivatives shown at right. Compute $\frac{\partial F}{\partial s}(2,1)$. (4 points)

\[
\begin{array}{|c|c|c|c|}
\hline
(x, y) & f(x, y) & \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\
\hline
(2,3) & 0 & 3 & 6 \\
(2,1) & 2 & -2 & -1 \\
(2,4) & 3 & 4 & 6 \\
(5,3) & 1 & 3 & 5 \\
\hline
\end{array}
\]

\[ \frac{\partial F}{\partial s}(2,1) = \]
**Extra credit problem:** Let $E : \mathbb{R}^2 \to \mathbb{R}$ be given by $E(x, y) = 2x + y^2$. Find a $\delta > 0$ so that $|E(h)| < 0.01$ for all $h = (x, y)$ with $|h| < \delta$. Carefully justify why the $\delta$ you provide is good enough.  

(2 points)