1. **Team Discussion**: Consider the two following differential equations:

\[
\frac{dy}{dx} = 4x \quad | \quad \frac{dy}{dx} = 4y
\]

(a) *Discuss*: What is a differential equation? Compare and contrast differential equations with equations you have seen before.

In a normal equation, you look for x-values satisfying the relation.

In differential eqs, you look for functions \( y \) that satisfy the relation involving the derivative.

(b) What is the difference between these two equations?

The first's derivative is a function of \( x \), and the second's derivative is a function of \( y \).

(c) What is a general form for the solution to each equation? (general \( C \) may include constants not yet defined)

\[
y = 2x^2 + C
\]
\[
\Rightarrow y' = 4x
\]
\[
y = Ce^{4x}
\]
\[
\Rightarrow y' = 4e^{4x} \cdot C = 4y
\]

(d) Find the solutions to the equations given that the graph of each solution goes through the point \((1, 12)\).

\[
12 = 2(1)^2 + C
\]
\[
10 = C
\]
\[
\Rightarrow y = 2x^2 + 10
\]
\[
12 = Ce^{4(1)}
\]
\[
\frac{12}{e^4} = C
\]
\[
\Rightarrow y = \frac{12}{e^4} e^{4x}
\]
\[
= 12 e^{-4} e^{4x}
\]
\[
= 12 e^{4x-4}
\]
2. A bullet is shot upward from the surface of a planet so that its height in meters until coming to rest is given by the equation \( s(t) = 100t - 5t^2 \) where \( t \) is measured in seconds. Answer the following questions and be sure to use proper units in each answer.

(a) Find equations for the bullet’s velocity and acceleration after \( t \) seconds.

\[
\begin{align*}
  s(t) &= 100t - 5t^2 \\
  v(t) &= s'(t) = 100 - 10t \\
  a(t) &= v'(t) = s''(t) = -10
\end{align*}
\]

(b) What is the acceleration due to gravity on this planet? If Earth has acceleration due to gravity approximately \(-9.8 \text{ m/s}^2\), does this planet have more or less mass than Earth?

\[ s = -10 \text{ b/c } a(t) = -10 \]

This is a slightly stronger acceleration, so the planet must have slightly more mass.

(c) What is the bullet’s initial velocity?

\[ v(0) = 100 - 10(0) = 100 \rightarrow 100 \text{ m/s} \]

(d) When is the bullet 375m high?

\[
\begin{align*}
  s(t) &= 375 = 100t - 5t^2 \\
  6t^2 - 100t + 375 &= 0 \\
  5(t^2 - 20t + 75) &= 0 \\
  t &= 5 \text{ sec and } 15 \text{ sec}
\end{align*}
\]

(e) What velocity is the bullet at these times?

\[
\begin{align*}
  v(5) &= 100 - 10(5) = 50 \text{ m/s} \\
  v(15) &= 100 - 10(15) = -50 \text{ m/s}
\end{align*}
\]

(f) How long does it take for the bullet to reach its maximum height?

\[
\begin{align*}
  \text{max height when } v(t) &= 0 . \\
  100 - 10t &= 0 \rightarrow t = 10 \\
  10(10 - t) &= 0 \rightarrow t = 10 \text{ sec.}
\end{align*}
\]
(g) What was the maximum height?

\[ s(10) = 100(10) - 5(10)^2 \]

\[ = 1000 - 500 = 500 \text{ m} \]

(h) When does the bullet hit the ground?

\[ s(t) = 0 = 100t - 5t^2 \]

\[ = 6(100 - 56) \quad \therefore t = 20 \]

\[ = 56(20-t) \quad \text{at 20 seconds} \]

(i) When was the bullet going the fastest?

*Bonus gun safety exploration question:*

What does this tell you about the dangers of firing a bullet straight up in the air?

\[ v(t) = 100 \quad \text{at } t = 0 \quad \therefore \text{ this is max speed because} \]

\[ = -100 \quad \text{at } t = 20. \]

\[ |v(t)| \leq 100 \quad \text{for } 0 \leq t \leq 20. \]

Thus when a bullet fired up hits the ground, it's going at the same speed as when it was fired. So don't do it! You don't know where it will land.

3. A curve passes through the point \((1, e^7)\) and has the property that for each point on the curve, the slope of the curve is equal to twice the \(y\)-coordinate. What is the equation of the curve?

\[
\frac{dy}{dx} = 2y \quad \Rightarrow \quad y = Ce^{2x}
\]

Plug in \((1, e^7)\)

\[ e^7 = Ce^{2(1)} \]

\[ e^7 = Ce^2 \]

\[ \therefore y = e^5 \cdot e^{2x} = e^{2x+5} \]

\[ \frac{e^7}{e^5} = C \]

\[ e^2 = C \]

\[ e^5 = C \]
4. Suppose that \( A \) represents the number of grams of a radioactive substance at time \( t \) seconds. Given that \( \frac{dA}{dt} = -0.7A \), how long does it take 20 grams of this substance to be reduced to 13 grams?

**Hint:** First find an equation for \( A \) in terms of \( t \), and then find the time \( t \) that the substance has 13 grams.

\[
\frac{dA}{dt} = -0.7A \Rightarrow A = Ce^{-0.7t} \\
A(0) = 20 \Rightarrow \frac{13}{20} = e^{-0.7t} \\
\Rightarrow A = 20e^{-0.7t} \\
\ln \left( \frac{13}{20} \right) = -0.7t \\
t = \frac{\ln (\frac{13}{20})}{-0.7} \text{ seconds}
\]

5. (Calculator Needed) Carbon-14 has a half-life of 5730 years.

(a) Suppose we had a 68-million-year-old dinosaur fossil. What fraction of the living dinosaur's carbon-14 would be remaining today?

\[
y(t) = y(0)e^{-kt} \\
y(5730) = 0.5y(0) \\
y(0)e^{-k(5730)} = \frac{1}{2}y(0) \\
k = \frac{\ln 0.5}{5730} \\
y(68,000,000) = y(0)e^{-68,000,000} \\
\approx y(0) \cdot 0 \text{ basically none}
\]

(b) Suppose the minimum detectable amount is 0.1%. What is the maximum age of a fossil that we could date using carbon-14?

\[
y(t) = (0.1\%)y(0) \\
0.001 = y(0)e^{-kt} \\
\ln 0.001 = -kt \\
t = \frac{-\ln 0.001}{k} \\
\approx 57,104 \text{ years}
\]

(c) Dinosaur fossils are often dated by using other isotopes, such as potassium-40, that has a longer half-life, approximately 1.25 billion years. What is the maximum age of a fossil we could date using potassium-40, assuming the minimum detectable amount is 0.1%?

Let \( k \) measure in millions of years.

\[
y(t) = y(0)e^{-kt} \\
y(1250) = 0.5y(0) \\
y(0)e^{-k(1250)} = \frac{1}{2}y(0) \\
e^{-k(1250)} = \frac{1}{2} \\
1250k = \ln \frac{1}{2} \\
k = -\frac{\ln \frac{1}{2}}{1250} \\
= \frac{\ln 2}{1250} \\
y(t) = (0.1\%)y(0) \\
0.001y(0) = y(0)e^{-kt} \\
t = \frac{-\ln 0.001}{k} = \frac{-\ln 0.001}{\frac{\ln 2}{1250}} \\
\approx 12,487 \text{ million years}, \\
\text{older than the Earth by } 3 \times
\]