1. The population of a town is currently 400, but it is expected to increase at a rate of $100e^{0.25t}$ people per year where $t$ represents the number of years from now. What is the population of this town expected to be in 12 years?

2. Evaluate the following definite integrals. Simplify each answer.

(a) $\int_{0}^{2} 10xe^{x^2} \, dx$

(b) $\int_{0}^{\pi/2} \sin^7 \left( x - \frac{\pi}{4} \right) \, dx$
3. Let $R$ be the finite region enclosed by $f(x) = x^4$ and $g(x) = 20 - x^2$. Sketch $R$ and label the points of intersection.

Using proper mathematical terminology, write down the definite integrals which represent the following quantities. Do not evaluate these integrals.

(a) The area of $R$.

(b) The volume of the solid obtained when $R$ is revolved around the line $x = 8$. Integrate with respect to $x$.

(c) The volume of the solid obtained when $R$ is revolved around the line $y = -2$. Integrate with respect to $x$.

(d) The volume of the solid with base $R$ for which the cross-sections perpendicular to the $x$-axis are squares.
4. Evaluate the following indefinite integrals.

(a) $\int x^7(x^4 - 3)^{50} \, dx$

(b) $\int \sin^3 x \cos^5 x \, dx$

(c) $\int \frac{1}{x^2 - 8x + 19} \, dx$
(d) \[ \int \frac{2x^7 + x^5 + 2x^2 + 2}{2x^2 + 1} \, dx \]

(e) \[ \int (\cos(2x) - 2\cos^2(x) - \tan^2(x)) \, dx \]

5. Suppose \( F(x) \) is a polynomial with \( F'(x) = f(x) \). Given that \( F(-1) = 4, F(1) = 7, F(3) = 23, F(5) = 34, \) and \( F(7) = 12 \), find the average value of \( f(x) \) on the interval \([-1, 5]\).