1. The population of a town is currently 400, but it is expected to increase at a rate of $100e^{0.25t}$ people per year where $t$ represents the number of years from now. What is the population of this town expected to be in 12 years?

\[
P_{\text{after 12 yrs}} = 400 + \int_0^{12} 100e^{0.25t} \, dt = 400 + \left[400e^{0.25t}\right]_0^{12} = 400 + 400e^3 - 400e^0 = 400e^3 \text{ people}
\]

2. Evaluate the following definite integrals. Simplify each answer.

(a) \[
\int_0^2 10xe^x \, dx = \int_0^4 5e^u \, du = \left[5e^u\right]_0^4 = 5e^4 - 5
\]

(b) \[
\int_0^{\pi/2} \sin^7\left(x - \frac{\pi}{4}\right) \, dx = \int_{-\pi/4}^{\pi/4} \sin^7(u) \, du
\]

$sin^7(u)$ is odd:
\[
\sin^7(-u) = (-\sin(u))^7 = -\sin^7(u)
\]

\[
= 0
\]
3. Let \( R \) be the finite region enclosed by \( f(x) = x^4 \) and \( g(x) = 20 - x^2 \). Sketch \( R \).

\[ x^4 = 20 - x^2 \]
\[ x^4 + x^2 - 20 = 0 \]
\[ (x^2 - 4)(x^2 + 5) = 0 \]
\[ x = \pm 2 \]

Using proper mathematical terminology, write down the definite integrals which represent the following quantities. Do not evaluate these integrals.

(a) The area of \( R \).

\[ A = \int_{-2}^{2} \left( (20 - x^2) - x^4 \right) \, dx \]

(b) The volume of the solid obtained when \( R \) is revolved around the line \( x = 8 \).

\[ \int_{-2}^{2} 2\pi \left( 8-x \right) \left( 20 - x^2 - x^4 \right) \, dx \]

(c) The volume of the solid obtained when \( R \) is revolved around the line \( y = -2 \).

\[ \int_{-2}^{2} \left( \pi \left( (20-x^2) - (-2) \right)^2 - \pi \left( x^4 - (-2) \right)^2 \right) \, dx \]

(d) The volume of the solid with base \( R \) for which the cross-sections perpendicular to the \( x \)-axis are squares.

\[ \int_{-2}^{2} \left( 20 - x^2 - x^4 \right)^2 \, dx \]
4. Evaluate the following indefinite integrals.

(a) \[ \int x^7(x^4 - 3)^{50} \, dx \quad u = x^4 - 3 \quad \Rightarrow \quad u^4 = u + 3 \]
\[ du = x^3 \, dx \]
\[ \frac{1}{4} du = x^3 \, dx \]
\[ = \frac{1}{4} \int \left( u^{51} + 3u^{50} \right) \, du = \frac{1}{4} \left( \frac{u^{52}}{52} + 3 \frac{u^{51}}{51} \right) + C \]
\[ = \frac{1}{4} \left( \frac{(x^4 - 3)^{52}}{52} + 3 \frac{(x^4 - 3)^{51}}{51} \right) + C \]

(b) \[ \int \sin^3 x \cos^5 x \, dx \]
\[ = \int \sin^2 x \cos^5 x \sin x \, dx \quad u = \cos x \]
\[ \quad du = -\sin x \, dx \]
\[ = -\int (1 - u^2)u^3 \, du = -\int (u^5 - u^3) \, du = \int (u^7 - u^5) \, du \]
\[ = \frac{1}{8} u^8 - \frac{1}{6} u^6 + C \]
\[ = \frac{1}{8} \cos^8 x - \frac{1}{6} \cos^6 x + C \]

(c) \[ \int \frac{1}{x^2 - 8x + 19} \, dx \]
\[ = \int \frac{1}{(x - 4)^2 + 1} \, dx \]
\[ = \frac{1}{3} \int \frac{1}{(x - 4)^2 + 1} \, dx \]
\[ = \frac{1}{3} \int \frac{1}{\left( \frac{x - 4}{\sqrt{3}} \right)^2 + 1} \, dx \]
\[ u = \frac{x - 4}{\sqrt{3}} \quad au = \frac{1}{\sqrt{3}} \, dx \]
\[ = \frac{\sqrt{3}}{3} \int \frac{1}{u^2 + 1} \, du = \frac{1}{\sqrt{3}} \arctan(u) + C \]
\[ = \frac{1}{\sqrt{3}} \arctan \left( \frac{x - 4}{\sqrt{3}} \right) + C \]
\[
\int \frac{2x^7 + x^5 + 2x^2 + 2}{2x^2 + 1} \, dx \quad \text{polynomial long division}\quad \frac{2x^7 + x^5 + 2x^2 + 2}{2x^2 + 1} = \frac{x^5 + 1}{2x^7 + x^5 + 2x^2 + 2} - \frac{2x^2 + 2}{2x^2 + 1}
\]

\[
= \int \left( x^5 + 1 + \frac{1}{2x^2 + 1} \right) \, dx = \int \left( x^5 + 1 \right) \, dx + \int \frac{1}{2x^2 + 1} \, dx
\]

\[
= \int \left( x^5 + 1 \right) \, dx + \int \frac{1}{(\sqrt{2}x)^2 + 1} \, dx, \quad u = \sqrt{2}x, \quad du = \sqrt{2} \, dx
\]

\[
= \int \left( x^5 + 1 \right) \, dx + \frac{1}{\sqrt{2}} \int \frac{1}{u^2 + 1} \, du = \frac{x^6}{6} + x + \frac{1}{\sqrt{2}} \arctan(u) + C
\]

\[
eq \frac{x^6}{6} + x + \frac{1}{\sqrt{2}} \arctan(\sqrt{2}x) + C
\]

\[
\int (\cos(2x) - 2 \cos^2(x) - \tan^2(x)) \, dx = \int (\cos^2(x) - \sin^2(x) - 1 - \cos^2(x) \tan^2(x)) \, dx
\]

\[
= \int (-1 - \tan^2(x)) \, dx = -\int (1 + \tan^2(x)) \, dx = -\int \sec^2(x) \, dx
\]

\[
= -\tan(x) + C
\]

5. Suppose \( F(x) \) is a polynomial with \( F'(x) = f(x) \). Given that \( F(-1) = 4, F(1) = 7, F(3) = 23, F(5) = 34, \) and \( F(7) = 12 \), find the average value of \( f(x) \) on the interval \([-1, 5]\).

\[
\text{avg} = \frac{1}{5-(-1)} \int_{-1}^{5} f(x) \, dx
\]

\[
= \frac{1}{6} \left( F(5) - F(-1) \right)
\]

\[
= \frac{1}{6} \left( 34 - 4 \right) = \frac{1}{6} \left( 30 \right) = 5
\]