1. Evaluate the limit
\[ \lim_{t \to 2} \frac{t^2 + t - 6}{t - 2} \]

Discuss the validity of the statement
\[ \frac{t^2 + t - 6}{t - 2} = t + 3 \]

**Answer:**
\[ \lim_{t \to 2} \frac{t^2 + t - 6}{t - 2} = \lim_{t \to 2} \frac{(t-2)(t+3)}{t - 2} = \lim_{t \to 2} t + 3 = 5 \]

Although the statement is algebraically true, the functions on either side of the equation are not the same function. The function on the left is not defined at \( t = 2 \), whereas \( t + 3 \) is definitely defined at \( t = 3 \).

2. Sketch the graph of a function \( f \) that satisfies these conditions: \( \lim_{x \to 0} f(x) = 1 \), \( \lim_{x \to 2} f(x) = \infty \), \( \lim_{x \to 2^+} f(x) = -1 \), \( f(1) = -4 \). What are the possible values of \( f(2) \)?

**Answer:** There are many possible graphs of such a function, and one example will not fully illustrate if your solution is correct. If you want to check your graph, come to office hours.

\( f(2) \) can be any real number, and it can be undefined.

3. For the function \( f \) whose graph is given, state the value of each quantity, if it exists. If it does not, explain why.

![Graph](image)

(a) \( \lim_{x \to 1} f(x) \) **Answer:** \( \lim_{x \to 1} f(x) = 2 \); as \( x \) approaches 1, the values of \( f(x) \) approach 2.

(b) \( \lim_{x \to 3^-} f(x) \) **Answer:** \( \lim_{x \to 3^-} f(x) = 1 \); as \( x \) approaches 3 from the left, the values of \( f(x) \) approach 1.
(c) \( \lim_{x \to 3^+} f(x) \) \textbf{Answer:} \( \lim_{x \to 3^+} f(x) = 4; \) as \( x \) approaches 3 from the right, the values of \( f(x) \) approach 4.

(d) \textbf{Answer:} \( \lim_{x \to 3} f(x) \) does not exist, because the limit from the left does not equal the limit from the right.

(e) \( f(3) \) \textbf{Answer:} \( f(3) = 3; \) when \( x = 3, \) \( y = 3. \)

4. Use tables to evaluate the following limits. If you know how to solve the limit a different way, do this to check your answer. (Tables can sometimes give you the wrong answer! Discuss why this is the case with your team.)

(a) \( \lim_{x \to 1} \frac{x^2 - 4}{x^2 + 2x - 8} \) \textbf{Answer:} Your table might look like:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \frac{x^2 - 4}{x^2 + 2x - 8} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.9</td>
<td>.5918</td>
</tr>
<tr>
<td>.99</td>
<td>.5992</td>
</tr>
<tr>
<td>.999</td>
<td>.5999</td>
</tr>
<tr>
<td>.001</td>
<td>.6001</td>
</tr>
<tr>
<td>.01</td>
<td>.6008</td>
</tr>
<tr>
<td>1</td>
<td>.6078</td>
</tr>
</tbody>
</table>

So it appears that \( \lim_{x \to 1} \frac{x^2 - 4}{x^2 + 2x - 8} = .6 \)

One can also solve this problem by finding the limit of the numerator and the limit of the denominator and dividing them:

(b) \( \lim_{x \to 2^+} \frac{x^2 - 2x - 8}{x^2 - 5x + 6} \) \textbf{Answer:} Your table might look like:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \frac{x^2 - 2x - 8}{x^2 - 5x + 6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>86.56</td>
</tr>
<tr>
<td>2.01</td>
<td>806.05</td>
</tr>
<tr>
<td>2.001</td>
<td>8006.01</td>
</tr>
</tbody>
</table>

So it appears that \( \lim_{x \to 2^+} \frac{x^2 - 2x - 8}{x^2 - 5x + 6} = \infty. \)

One can also solve this problem by factoring the numerator and denominator:

\[
\lim_{x \to 2^+} \frac{x^2 - 2x - 8}{x^2 - 5x + 6} = \lim_{x \to 2^+} \frac{(x - 4)(x + 2)}{(x - 3)(x - 2)}
\]

Since the numerator is negative when \( x \) is close to 2, and the denominator approaches 0 through negative values as \( x \to 2^+ \), the answer is positive infinity.
(c) \( \lim_{x \to 3^-} \frac{\ln(e^x)}{x-3} \) **Answer:** First note \( \ln(e^x) = x \). So we want to solve \( \lim_{x \to 3^-} \frac{x}{x-3} \).

Your table might look like:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \frac{x}{x-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9</td>
<td>-29</td>
</tr>
<tr>
<td>2.99</td>
<td>-299</td>
</tr>
<tr>
<td>2.999</td>
<td>-2999</td>
</tr>
</tbody>
</table>

So it appears that \( \lim_{x \to 3^-} \frac{x}{x-3} = -\infty \)

One can solve this problem by saying that the numerator is positive and the denominator approaches 0 through negative values as \( x \to 3^- \).

(d) \( \lim_{x \to 0^+} \ln(\sin x) \)

Does your table for this one look different than the previous table? Talk with your team about why this might be the case.

**Answer:** Your table might look like:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \ln(\sin x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-2.3</td>
</tr>
<tr>
<td>0.01</td>
<td>-4.61</td>
</tr>
<tr>
<td>0.001</td>
<td>-6.91</td>
</tr>
<tr>
<td>0.000000000000001</td>
<td>-34.54</td>
</tr>
</tbody>
</table>

So it appears that \( \lim_{x \to 0^+} \ln(\sin x) = -\infty \).

This function approaches infinity approximately linearly as you divide by 10 because of the logarithm.

One can also solve this problem by deducing what the sine function does: \( \sin x \to 0^+ \) as \( x \to 0^+ \), and \( \ln(t) \to -\infty \) as \( t \to 0^+ \).

5. Evaluate the limits without using tables and explain your reasoning.

(a) \( \lim_{x \to 2} ax^2 + bx + c + \log_2(x) \) **Answer:** \( \lim_{x \to 2} x^2 = 4, \ lim_{x \to 2} x = 2, \ \text{and} \ \lim_{x \to 2} \log_2(x) = 1, \) so \( \lim_{x \to 2} ax^2 + bx + c + \log_2(x) = 4a + 2b + c + 1 \).

(b) \( \lim_{x \to 5} \frac{x^2 - 16}{x^2 - x - 12} \) **Answer:** \( \lim_{x \to 5} x^2 - 16 = 9, \ \text{and} \ \lim_{x \to 5} x^2 - x - 12 = 8, \) so \( \lim_{x \to 5} \frac{x^2 - 16}{x^2 - x - 12} = \frac{9}{8} \)

(c) \( \lim_{x \to 4} \frac{x^2 - 16}{x^2 - x - 12} \) **Answer:** \( \lim_{x \to 4} \frac{x^2 - 16}{x^2 - x - 12} \to 0 \) This is an indeterminate form, so we must do more work.

\[
\lim_{x \to 4} \frac{x^2 - 16}{x^2 - x - 12} = \lim_{x \to 4} \frac{(x - 4)(x + 4)}{(x - 4)(x + 3)} = \lim_{x \to 4} \frac{x + 4}{x + 3} = \frac{4 + 4}{4 + 3} = \frac{8}{7}
\]
(d) \( \lim_{x \to 0} \left( \frac{1}{3x} - \frac{4}{x^2 + 12x} \right) \)  

**Answer:**

\[
\begin{align*}
\lim_{x \to 0} \left( \frac{1}{3x} - \frac{4}{x^2 + 12x} \right) &= \lim_{x \to 0} \left( \frac{1}{3x} - \frac{4}{x(x + 12)} \right) \\
&= \lim_{x \to 0} \left( \frac{1}{3x} \cdot \frac{x + 12}{x + 12} - \frac{4}{x(x + 12)} \cdot \frac{3}{3} \right) \\
&= \lim_{x \to 0} \left( \frac{x + 12}{3x(x + 12)} - \frac{4 \cdot 3}{3x(x + 12)} \right) \\
&= \lim_{x \to 0} \left( \frac{x}{3x(x + 12)} \right) \\
&= \lim_{x \to 0} \left( \frac{1}{3(x + 12)} \right) \\
&= \frac{1}{3(0 + 12)} = \frac{1}{3 \cdot 12} = \frac{1}{36}
\end{align*}
\]

**Review**

1. Sketch the function \( f(x) = \log_2(x + 4) \).

**Answer:** This function is \( \log_2(x) \) shifted left four units.

(a) Does \( f(x) \) intersect \( g(x) = 2^x \)? How do you know?

**Answer:** This function does intersect \( g(x) \). One way to know is that \( f(0) = \log_2(4) = 2 \), and \( g(0) = 2^0 = 1 \), and \( f \) curves down towards negative infinity, so they must intersect.

(b) Find \( f^{-1}(x) \).

**Answer:**

\[
\begin{align*}
y &= \log_2(x + 4) \\
2^y &= x + 4 \\
2^y - 4 &= x
\end{align*}
\]

So \( f^{-1}(x) = 2^x - 4 \).

(c) Does \( f^{-1}(x) \) intersect \( g(x) \)? Does it intersect \( e^x \)? How do you know?

**Answer:** This function does not intersect \( g(x) \) - it is the same function, but shifted down. For every value of \( x \), \( f^{-1}(x) = g(x) - 4 \).

It also does not intersect \( e^x \), because \( e \approx 2.17 \), so this function will grow faster than \( f^{-1} \) in the first quadrant, and is not negative when \( x \) is negative, unlike \( f^{-1} \).
2. True or false?

- \( \ln(ab) = \ln(a) + \ln(b) \) **Answer:** True
- \( \ln(ab) = \ln(a) \ln(b) \) **Answer:** False
- \( \ln(a) - \ln(b) = \frac{\ln(a)}{\ln(b)} \) **Answer:** False
- \( \frac{\ln(a)}{\ln(b)} = \ln\left(\frac{a}{b}\right) \) **Answer:** False
- \( \ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right) \) **Answer:** True
- \( \ln(e) = 1 \) **Answer:** True
- \( \ln(0) = 1 \) **Answer:** False, because \( \ln(0) \) does not exist.
- \( \ln(1) = 0 \) **Answer:** True
- \( \ln\left(x^a\right) = a \ln\left(x\right) \) **Answer:** True
- \( \ln\left(5x^a\right) = 5a \ln\left(x\right) \) **Answer:** False
- \( \ln\left(5x^a\right) = a \ln\left(x\right) \) **Answer:** False
- \( \ln(e^x) = x \) **Answer:** True
- \( \ln\left(5e^x\right) = 5x \) **Answer:** False
- \( e^{\ln(x)} = x \) **Answer:** True

3. Determine all values of \( x \) which satisfy the equation \( \ln\left(x^7\right) = 35 \).

**Answer:**

\[
\ln\left(x^7\right) = 35 \\
7 \ln x = 35 \\
\ln x = 5 \\
e^{\ln(x)} = e^5 \\
x = e^5
\]