1. (4 points) A cone-shaped vat with diameter 10 ft and height 15 ft contains a liquid that is draining out through a hole in the bottom at a constant rate of 2 cubic feet per second. How quickly is the height of the liquid decreasing when its height is 3 ft?

\[
\begin{align*}
\text{radius} &= r \text{ (ft)} \\
\text{height} &= h \text{ (ft)} \\
\text{volume} &= V \text{ (ft}^3\text{)} \\
\text{time} &= t \text{ (sec)}
\end{align*}
\]

\[V = \frac{\pi r^2 h}{3}\]

by similar triangles,

\[\frac{5}{16} = \frac{r}{h} \Rightarrow \frac{1}{3} = \frac{r}{h}\]

\[\Rightarrow 3r = h \Rightarrow r = \frac{1}{3}h\]

\[\Rightarrow V = \frac{\pi}{3} \left(\frac{h}{3}\right)^2 \frac{h}{3}\]

\[V = \frac{\pi h^3}{27}\]

Implicit differentiation:

\[\frac{dV}{dt} = \frac{\pi}{3} \cdot \frac{3h^2}{27} \frac{dh}{dt}\]

\[-2 = \frac{\pi}{27} \cdot 3 \cdot 9 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = -\frac{2}{\pi}\]

decreasing at \(-\frac{2}{\pi}\) ft/sec
2. (3 points) Suppose that $g$ and $g'$ are differentiable everywhere and satisfy the following conditions.

- $g(2) = 8$
- $g'(2) = 10$
- $g''(2) = 2$

Use a linear approximation to estimate the following quantities. Simplify and write your answers in decimal form.

(a) $g(1.9)$

$$g(x) \approx L_1(x) = g(2) + g'(2)(x-2)$$

$$= 8 + 10(x-2)$$

@ $x=1.9$: $L_1(x) = 8 + 10(-.1) = 8 + (-1) = 7$

(b) $g'(1.9)$

$$g'(x) \approx L_2(x) = g'(2) + g''(2)(x-2)$$

$$= 10 + 2(x-2)$$

@ $x=1.9$: $L_2(x) = 10 + 2(-.1) = 10 + (-.2) = 9.8$

3. (3 points) Use a linear approximation to obtain a good estimate for $\sqrt{100004}$. Simplify and write your answer in decimal form.

$$f(x) = \sqrt{x^4}, \ a = 10000$$

$$f(x) \approx L(x) = f(a) + f'(a)(x-a)$$

$$f(10000) = \sqrt{100000} = 10$$

$$f'(x) = \frac{1}{4} x^{-3/4}$$

$$f'(10000) = \frac{1}{4} (10000)^{-3/4} = \frac{1}{4(\sqrt[4]{10000})^3} = \frac{1}{4(10)^3} = \frac{1}{4000}$$

$$f(x) \approx L(x) = 10 + \frac{1}{4000}(x-10000)$$

@ $x=10004$:

$$f(10004) \approx L(10004) = 10 + \frac{1}{4000}(4) = 10 + \frac{1}{1000} = 10.001$$