Let $R$ be the finite region bounded by the graphs of $y = \sqrt{x}$, $x = 4$, and the $x$-axis. Set up, but do not evaluate, definite integrals which represent the volumes of the following solids.

1. (4 points) The volume of the solid with base $R$ for which the cross-sections perpendicular to the $x$-axis are semi-circles.

   \[
   \text{semi-circle radius } r = \frac{\sqrt{x}}{2}
   \]

   for $0 \leq x \leq 4$

   cross-sect area is $\frac{1}{2} \pi r^2 = \frac{1}{2} \pi \left(\frac{\sqrt{x}}{2}\right)^2 = \frac{1}{8} \pi x$

   integral is

   \[
   \int_0^4 \frac{1}{8} \pi x \, dx = \frac{1}{8} \pi \int_0^4 x \, dx
   \]

2. (2 points) The volume of the solid formed when $R$ is revolved around the line $y = 5$. Integrate with respect to $x$.

   \[
   \text{WASHER METHOD}
   \]

   inner rad = $5 - \sqrt{x}$

   outer rad = 5

   $A(x) = \pi (5)^2 - \pi (5-\sqrt{x})^2$

   integral is:

   \[
   \int_0^4 \pi (5)^2 - \pi (5-\sqrt{x})^2 \, dx
   \]

   \[
   = \pi \int_0^4 25 - (5-\sqrt{x})^2 \, dx
   \]
3. (2 points) The volume of the solid formed when \( R \) is revolved around the line \( x = -1 \). Integrate with respect to \( x \).

CYLINDRICAL SHELLS

height of cylinder: \( \sqrt{x} \)

radius of cylinder: \( x+1 \)

\[ V = \int_{0}^{4} 2\pi \,(x+1)\,(\sqrt{x}) \,dx \]

4. (2 points) The volume of the solid formed when \( R \) is revolved around the line \( x = -1 \). Integrate with respect to \( y \).

WASHERS

inner rad = \( 1+y^2 \)

outer rad = \( 5 \)

\( A(x) = \pi \,(5)^2 - \pi \,(1+y^2)^2 \)

\[ V = \int_{0}^{2} \pi \,(5)^2 - \pi \,(1+y^2)^2 \,dy \]

\[ = \pi \int_{0}^{2} 25 - (1+y^2)^2 \,dy \]