Name ____________________________  NetID: ______________________
UIN: __________________________

• 50 minutes  • No calculators  • Show sufficient work

• Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
• Remove hats and sunglasses.
• You must show sufficient work to justify each answer.
• While the test is in progress, we will not answer questions concerning the test material.
• Do not leave early unless you are at the end of a row.

100 points total.
1. (10 points each) Evaluate the indefinite integrals.

(a) \[ \int \tan^3 x \sec^5 x \, dx \]

(b) \[ \int x^{13} (x^7 + 9)^{98} \, dx \]
(c) \[ \int \frac{x^9 + x^7 + 42}{x^2 + 1} \, dx \]

(d) \[ \int 72e^{9x} \csc^2(e^{9x}) \, dx \]
(e) $\int \cos^2(6x) \sin(3x) \sin(6x) \, dx$
2. (10 points) Use integration to find the area of the finite region in the first quadrant bounded by the straight line which intersects the $y$-axis at $y = h$ and the $x$-axis at $x = b$, where $h$ and $b$ are arbitrary positive numbers.
3. (10 points) Let $R$ be the finite region bounded by the graphs of $y = x^2 + 1$ and $y = 4x - 2$. Set up, but do not evaluate, a definite integral which represents the volume of the solid with base $R$ whose cross-sections parallel to the $x$-axis are semi-circles.
4. (10 points) Let $R$ be the finite region in the first quadrant which is bounded by the graphs of $y = \sqrt{x}$, $y = x - 6$, and the $x$-axis. In the following manner, set up, but do not evaluate, definite integrals which represent the volume of the solid obtained when $R$ is revolved about the vertical line $x = 15$. Use proper notation.

(a) Integrate with respect to $x$.

(b) Integrate with respect to $y$. 
5. (10 points) Water flows from the bottom of a storage tank at a rate of $200 - 4t$ liters per minute, where $0 \leq t \leq 50$. If the tank started with 3000 liters, how much water is left in the tank after the first 10 minutes?
6. (10 points) Find the average value of the function $f(x) = \frac{32x}{\sqrt{2x^2 + 49}}$ on the interval [0, 4]. Simplify your answer.