Crocheting Hyperbolic Surfaces

Girls Engaged in Math and Science Workshop

University of Illinois, at Urbana-Champaign

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Quote (Confucius 555-479 BC)

What I hear I forget,
What I see I remember,
What I touch I understand.
Curvature

Curvature is a measure of instantaneously how much a curve bends per unit length.

Formally for a curve in the plane, curvature is defined as

\[ \kappa = \left| \frac{dT}{ds} \right| \]

where \( s \) is the arc length and \( T \) is the unit tangent.
Zero Curvature
Positive Curvature
Negative Curvature
Maps

- Mercator Projection
- Gall-Peters Projection
- Miller Cylindrical Projection
- Mollweide Projection
- Goode's Homolosine Equal-area Projection
- Sinusoidal Equal-Area Projection
- Robinson Projection
Visualizing Hyperbolic Geometry

Poincaré disc model of the hyperbolic plane

\[ \left\{ x \in \mathbb{R}^2 : |x| < 1 \right\} \text{ with hyperbolic metric } ds^2 = \frac{dx^2 + dy^2}{(1 - x^2 - y^2)^2} \]
Visualizing Hyperbolic Geometry

Objects near the edge of the Poincaré disc are larger than they appear!
Hyperbolic Geometry in Nature
Paper Hyperbolic Plane
Crocheting Hyperbolic Surfaces

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Crocheted Hyperbolic Plane
Instructions for the Symmetric Hyperbolic Plane

1. Chain 3 stitches and join

2. Pick a constant $k$ ($k = 5$ works well)

3. Single crochet $k$ stitches

4. Increase

5. Repeat steps 3 and 4
Thank you for listening!