These problems are due on Wednesday, October 21st in class, and will be graded on clarity of exposition as well as correctness. If you work in a group, please write the names of all the members on your homework sheet.

The problem set this week is 13.7, 13.37, 13.23, 14.8, 14.14 and the following:

1. Let $\mathcal{P}(\mathbb{N})$ denote the power set of $\mathbb{N}$, i.e., elements of $\mathcal{P}(\mathbb{N})$ are subsets $S \subset \mathbb{N}$. Adapt the “diagonal argument” to prove that $\mathcal{P}(\mathbb{N})$ is uncountable.

2. Define a sequence by $x_1 = a$ and $x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right)$. Prove this sequence has a limit $L$ and that $L^2 = a$ without assuming that square roots exist, i.e., use this to give an alternate prove square roots exist. HINT: If you can prove that $x_n^2 \geq a$ for all $n$, you can use it to show that the sequence is decreasing. (This is called Newton’s method.)