These problems are due on Wednesday, December 2\textsuperscript{nd} in class, and will be graded on clarity of exposition as well as correctness. If you work in a group, please write the names of all the members on your homework sheet.

The problem set this week is 7.49 and 7.53 from Chapter 7 and the following problems.

1. Fix a natural number $n > 1$ and consider $\mathbb{Z}/n\mathbb{Z}$ as a group under addition. We have discussed in class the map $\mathbb{Z}/n\mathbb{Z} \to \mathbb{Z}/n\mathbb{Z}$ that multiplies $\overline{k}$ by a fixed $a$. Is this map a group homomorphism? What is its kernel? (HINT: Use the lemma we proved in class relating the gcd of $a$ and $n$ to the existence of a multiplicative inverse for $a$.)

2. (a) Let $p$ be a prime and $a$ be any integer. Prove that

$$\text{If } \ k \equiv 1 \mod (p-1) \text{ then } a^k \equiv a \mod p.$$  

(b) Let $p$ and $q$ be distinct primes and $a$ be any integer. Prove that

$$\text{If } \ k \equiv 1 \mod (p-1)(q-1) \text{ then } a^k \equiv a \mod pq.$$