# Solution for Nov. 29

5.5

62 $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^5\theta \, d\theta$

Method 一: $\sin\theta$ is odd function, so $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^5\theta \, d\theta = 0$

Method 二:

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^5\theta \, d\theta$$

$$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (1-\cos^2\theta)^2 \sin\theta \, d\theta$$

$$= -\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (1 - 2\cos^2\theta + \cos^4\theta) \, d(\cos\theta)$$

$$= -\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} d(\cos\theta) + 2\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos^2\theta \, d(\cos\theta) - \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos^4\theta \, d(\cos\theta)$$

$$= -\cos\theta \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} + \frac{2}{3}\cos^3\theta \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} - \frac{1}{5}\cos^5\theta \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

$$= 0$$

64 $\int_0^4 \frac{x}{\sqrt{1+2x}} \, dx$

$$= \frac{1}{2}\int_0^4 \frac{2x+1-1}{\sqrt{1+2x}} \, dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{1+2x} \, dx - \frac{1}{2}\int_0^4 \frac{1}{\sqrt{1+2x}} \, dx$$

$$= \frac{1}{4}\int_0^4 (1+2x)^{\frac{1}{2}} d(1+2x) - \frac{1}{4}\int_0^4 (1+2x)^{-\frac{1}{2}} d(1+2x)$$

$$= \frac{1}{4} \cdot \frac{1}{1+\frac{1}{2}} (1+2x)^{1+\frac{1}{2}} \Big|_0^4 - \frac{1}{2}(1+2x)^{\frac{1}{2}} \Big|_0^4$$

$$= \frac{10}{3}$$

66 $\int_0^{\frac{1}{2}} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

Let $u = \sin^{-1}x$, then $du = \frac{1}{\sqrt{1-x^2}}\,dx$

$$\int_0^{\frac{1}{2}} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int_0^{\frac{\pi}{6}} u\,du = \frac{1}{2}u^2\Big|_0^{\frac{\pi}{6}} = \frac{\pi^2}{72}$$

6.1

2. $A = \int_0^6 [2x-(x^2-4x)]\,dx = 3x^2\Big|_0^6 - \frac{1}{3}x^3\Big|_0^6 = 36$

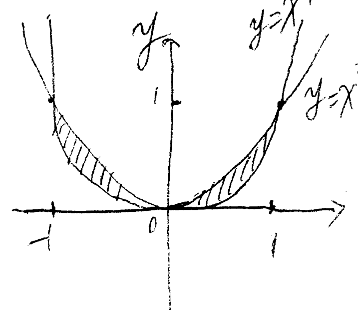
4 $A = \int_{-1}^{2} [y^2-(y-5)]\,dy = \frac{1}{3}y^3\Big|_{-1}^{2} - \frac{1}{2}y^2\Big|_{-1}^{2} + 5y\Big|_{-1}^{2} = 16.5$

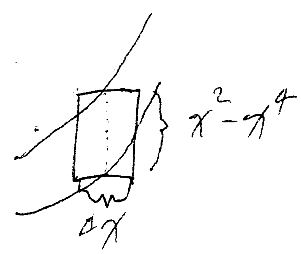
8. $A = \int_{-1}^{1} (x^2-x^4)\,dx$

$= 2\int_0^1 (x^2-x^4)\,dx$

$= 2\left(\frac{1}{3}x^3-\frac{1}{5}x^5\right)\Big|_0^1$

$= \frac{4}{15}$

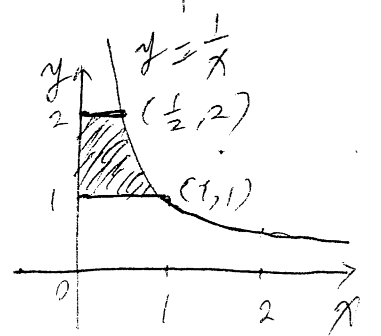


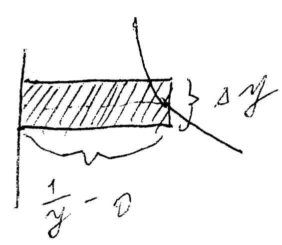




18 $A = \int_1^2 \left(\frac{1}{y}\right) dy$

$= \ln y\Big|_1^2$

$= \ln 2$

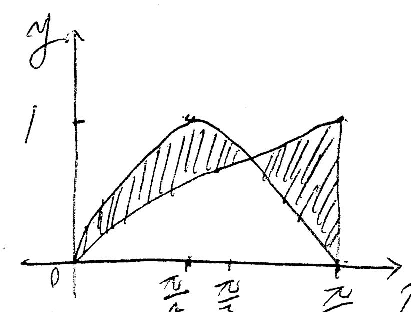



22 Solving $\sin x = \sin 2x = 2\sin x\cos x$

$\sin x = 0$ or $\cos x = \frac{1}{2}$

so $x = 0$ or $x = \frac{\pi}{3}$




$$A = \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x)\,dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\sin x - \sin 2x)\,dx$$

$$= \left(-\frac{1}{2}\cos 2x + \cos x\right)\Big|_0^{\frac{\pi}{3}} + \left(\frac{1}{2}\cos 2x - \cos x\right)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}$$

28 $A = \int_0^2 \left[ \left(-\frac{4}{5}x+5\right) - \left(-\frac{7}{2}x+5\right) \right] dx$

$+ \int_2^5 \left[ \left(-\frac{4}{5}x+5\right) - (x-4) \right] dx$

$= \int_0^2 \left(\frac{27}{10}x\right) dx + \int_2^5 \left(-\frac{9}{5}x+9\right) dx$

$= \frac{27}{20}x^2 \Big|_0^2 + \left(-\frac{9}{10}x^2+9x\right)\Big|_2^5$

$= \frac{27}{2}$