4.4

2 a) $\lim_{x\to a}[f(x)p(x)]$ is an indeterminate form of type $0\cdot\infty$.

b). When x is near a, $p(x)$ and $q(x)$ are both large, so $p(x)q(x)$ is large. Thus, $\lim_{x\to a}[h(x)p(x)]=\infty$.

c). When x is near a, $f(x)$ is near 0, $p(x)$ is large, so $f(x)-p(x)$ is large negative. Thus, $\lim_{x\to a}[f(x)-p(x)]=-\infty$.

10. $\lim_{x\to 0}\frac{x+\tan x}{\sin x}=\lim_{x\to 0}\frac{1+\sec^2 x}{\cos x}=\frac{1+1^2}{1}=2$

12. $\lim_{x\to\pi}\frac{\tan x}{x}=\frac{\tan\pi}{\pi}=\frac{0}{\pi}=0.$

14. $\lim_{x\to\frac{3\pi}{2}}\frac{\cos x}{x-\frac{3\pi}{2}}\overset{H}{=}\lim_{x\to\frac{3\pi}{2}}\frac{-\sin x}{1}=-\sin\frac{3\pi}{2}=1.$

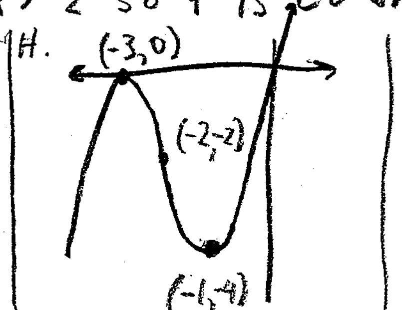
42. $\lim_{x\to(\frac{\pi}{2})^-}(\sec 7x\cos 3x)=\lim_{x\to(\frac{\pi}{2})^-}\frac{\cos 3x}{\cos 7x}=\lim_{x\to(\frac{\pi}{2})^-}\frac{-3\sin 3x}{-7\sin 7x}=\frac{3(-1)}{7(-1)}=\frac{3}{7}$

50. $\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)=\lim_{x\to 1}\left(\frac{x-1-\ln x}{(x-1)\ln x}\right)\overset{H}{=}\lim_{x\to 1}\frac{1-\frac{1}{x}}{\ln x+(x-1)\cdot\frac{1}{x}}$

$=\lim_{x\to 1}\left(\frac{x-1}{x\ln x+(x-1)}\right)\overset{H}{=}\lim_{x\to 1}\left(\frac{1}{\ln x+1+1}\right)=\frac{1}{0+2}=\frac{1}{2}.$

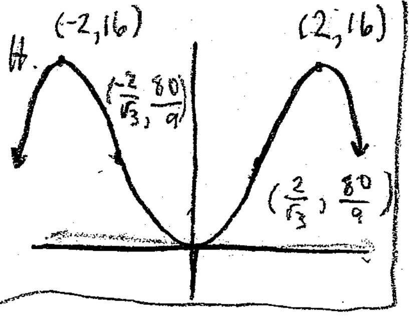
4.5

2. $y=f(x)=x^3+6x^2+9x=x(x+3)^2$ A. $D=\mathbb{R}$ B. x-intercepts are $-3, 0$. y-intercept $=0$.
C. no symmetry D. no asymptote E. $f'(x)=3x^2+12x+9=3(x+1)(x+3)<0\Rightarrow$ $-3<x<-1$, f decreasing $(-3,1)$ and increasing on $(-\infty,-3)$ and $(-1,\infty)$.
F. local maximum $f(-3)=0$, local minimum $f(-1)=-4$
G. $f''(x)=6x+12=6(x+2)>0\Rightarrow x>-2$ so f is CU on $(-2,\infty]$ and CD on $(-\infty,-2)$. IP at $(-2,-2)$. H.



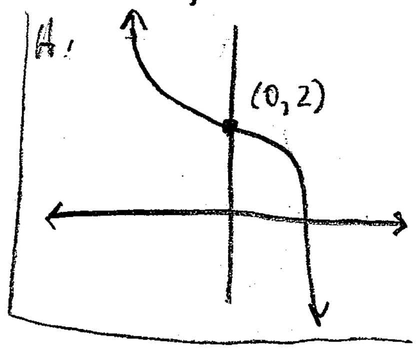
4. $y=f(x)=8x^2-x^4=x^2(8-x^2)$ A. $D=\mathbb{R}$
B. y-intercept $f(0)=0$ x-intercepts $f(x)=0$ $\Rightarrow x=0, \pm2\sqrt{2}$ C. $f(-x)=f(x)$ so f even and symmetric about the y-axis
D. no asymptote E. $f'(x)=16x-4x^3=4x(4-x^2)=4x(2+x)(2-x)>0\Rightarrow$ $x<-2$ or $0<x<2$ so f is increasing on $(-\infty,-2)$ and $(0,2)$ and decreasing on $(-2,0)$ and $(2,\infty)$. F local maxima $f(\pm2)=16$ local minimum $f(0)=0$ G $f''(x)=16-12x^2=4(4-3x^2)=0\Rightarrow x=\pm\frac{2}{\sqrt{3}}$, $f''(x)>0$ $\Rightarrow -\frac{2}{\sqrt{3}}<x<\frac{2}{\sqrt{3}}$ so f is CU on $\left(\frac{-2}{\sqrt{3}},\frac{2}{\sqrt{3}}\right)$ and CD on $\left(-\infty,\frac{-2}{\sqrt{3}}\right)$ and $\left(\frac{2}{\sqrt{3}},\infty\right)$. IP at $\left(\pm\frac{2}{\sqrt{3}},\frac{80}{9}\right)$.

4. H.



⑥ $y = f(x) = 2 - x - x^9 = -(x-1)(x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+2)$ A. $D = \mathbb{R}$ B y-intercept $f(0)=2$; x-intercept $f(x)=0 \Rightarrow x=1$ C. no symmetry D. No asymptote E. $f'(x) = -1-9x^8 = -1(9x^8+1) < 0$ for all x, so f is decreasing on $\mathbb{R}$.

F. There is no extremum. G $f''(x) = -72x^7 > 0 \Rightarrow x<0$, so f is CU on $(-\infty, 0)$ and CD on $(0,\infty)$ IP at $(0,2)$ H.



⑧ $y = \frac{x}{(x-1)^2}$ A. $D = \{x \mid x \neq 1\}$

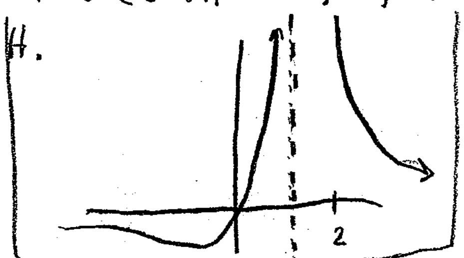
B. x-intercept $=0$, y-intercept $= f(0) = 0$.

C. no symmetry D. $\lim_{x\to\pm\infty} \frac{x}{(x-1)^2} = 0$ so $y=0$ is a HA. $\lim_{x\to 1} \frac{x}{(x-1)^2} = \infty$ so $x=1$ is a VA.

E. $f'(x) = \frac{(x-1)^2 \cdot 1 - x \cdot 2 \cdot (x-1)}{(x-1)^4} = \frac{-x-1}{(x-1)^3}$. $f'(x) < 0$ on $(-\infty,-1)$ and $(1,\infty)$

$f'(x) > 0$ on $(-1,1)$ so $f(x)$ is decreasing on $(-\infty,-1)$ and $(1,\infty)$ and increasing on $(-1,1)$. F local min $f(-1) = \frac{-1}{4}$, no local max.

G. $f''(x) = \frac{(x-1)^3 \cdot -1 + (x+1) \cdot 3(x-1)^2}{(x-1)^6} = \frac{2(x+2)}{(x-1)^4}$. This is negative on $(-\infty,-2)$ and positive on $(-2,1)$ and $(1,\infty)$. f is CD on $(-\infty,-2)$ and CU on $(-2,1)$ and $(1,\infty)$. IP at $(-2, \frac{-2}{9})$. H.



⑩ $f(x) = \frac{x}{x^2-9}$

C. $f(-x) = -f(x)$ so f odd; symmetric about the origin.

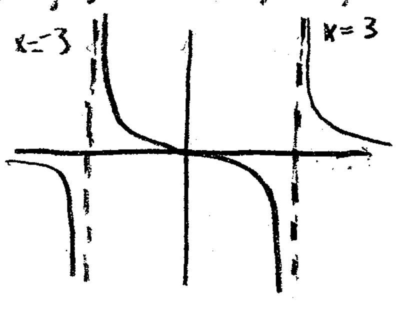
A. $D = \{x \mid x \neq \pm 3\}$

B. x-intercept $=0$, y intercept $=0$

D. $\lim_{x\to\pm3^+} \frac{x}{x^2-9} = \infty$, $\lim_{x\to\pm3^-} \frac{x}{x^2-9} = -\infty$, so $x=3$ and $x=-3$ are VA.

$\lim_{x\to\pm\infty} \frac{x}{x^2-9} = 0$ so $y=0$ is a HA. E. $f'(x) = \frac{(x^2-9) - x(2x)}{(x^2-9)^2} = -\frac{x^2+9}{(x^2-9)^2} < 0$ with $x \neq \pm 3$ so f decreasing on $(-\infty,-3), (-3,3), (3,\infty)$ F. no extremum.

G. $f''(x) = \frac{2x(x^2-9)^2 - (x^2+9) \cdot 2(x^2-9)(2x)}{(x^2-9)^4} = \frac{2x(x^2+27)}{(x^2-9)^3} > 0$ when $-3 < x < 0$ or $x > 3$, so f is CU on $(-3,0)$ and $(3,\infty)$; CD on $(-\infty,-3)$ and $(0,3)$. IP is $(0,0)$. H.



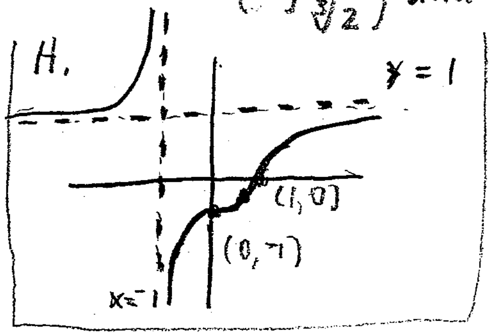
(16) $y=f(x)=\dfrac{x^3-1}{x^3+1}$ A. $D=\{x \mid x\neq -1\}$ B. x-intercept $=1$, y-intercept $=-1$

C. no symmetry D. $\lim\limits_{x\to\pm\infty}\dfrac{x^3-1}{x^3+1}=\lim\limits_{x\to\pm\infty}\dfrac{1-\frac{1}{x^3}}{1+\frac{1}{x^3}}=1$ so $y=1$ is a HA.

$\lim\limits_{x\to -1^-}\dfrac{x^3-1}{x^3+1}=\infty$ and $\lim\limits_{x\to -1^+}\dfrac{x^3-1}{x^3+1}=-\infty$, so $x=-1$ is a VA.

E. $f'(x)=\dfrac{(x^3+1)\,3x^2-(x^3-1)(3x^2)}{(x^3+1)^2}=\dfrac{6x^2}{(x^3+1)^2}>0$, with $x\neq -1$ so f is increasing on $(-\infty,-1)$ and $(-1,\infty)$. F. No extremum.

G. $y''=\dfrac{12x(x^3+1)^2-6x^2\cdot 2(x^3+1)\cdot 3x^2}{(x^3+1)^4}=\dfrac{12x(1-2x^3)}{(x^3+1)^3}>0 \Rightarrow x<-1$ or $0<x<\frac{1}{\sqrt[3]{2}}$ so f is CU on $(-\infty,-1)$ and $(0,\frac{1}{\sqrt[3]{2}})$ and CD on $(-1,0)$ and $(\frac{1}{\sqrt[3]{2}},\infty)$. IP $(0,-1)$, $(\frac{1}{\sqrt[3]{2}},-\frac{1}{3})$. H.



4.7 (2) The 2 numbers are x and $100+x$. $f(x)=(x+100)\cdot x=x^2+100x$. Minimize $f(x)$:

$f'(x)=2x+100=0 \Rightarrow x=-50$

The two numbers are 50 and -50.

(4) $x>0$ and $f(x)=x+\frac{1}{x}$. Minimize $f(x)$: $f'(x)=1-\frac{1}{x^2}=\frac{1}{x^2}(x^2-1)=\frac{1}{x^2}(x+1)(x-1)$ so the only crit. # in $(0,\infty)$ is 1. $f'(x)<0$ for $0<x<1$ and $f'(x)>0$ for $x>1$, so f has abs. minimum at $x=1$ and $f(1)=2$.

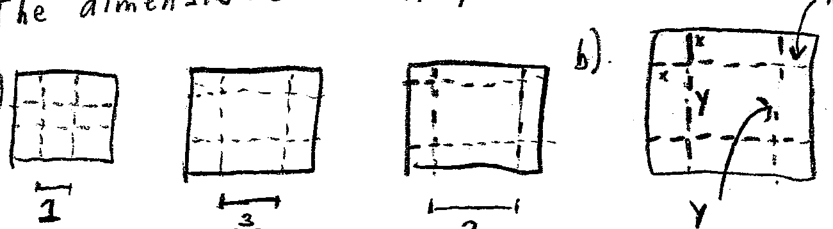
(6) Area $=x\cdot y=1000$ $y=\frac{1000}{x}$. Perimeter $P=2x+2y=2x+\frac{2000}{x}$. Minimize $P(x)$:

$P'(x)=2-\dfrac{2000}{x^2}=\dfrac{2}{x^2}(x^2-1000)$, so only crit. # in domain is $x=\sqrt{1000}$

$P''(x)=\dfrac{4000}{x^3}>0$, so P is CU and $P(\sqrt{1000})=4\sqrt{1000}$ is an abs. minimum.

The dimensions are $x=y=10\sqrt{10}$ m.

(8) a) b).



c). $V=y\cdot y\cdot x=xy^2$

d). $x+y+x=3$ $y+2x=3$

e). $y+2x=3$ $y=3-2x$

$V(x)=x(3-2x)^2$

$V=1$ $V=1.6875$ $V=2$

f). $V(x)=x(4x^2-12x+9)=4x^3-12x^2+9x \Rightarrow$

$V'(x)=3(4x^2-8x+3)=3(2x-1)(2x-3)$, so the crit. #s are $x=\frac{1}{2}$ and $x=\frac{3}{2}$. $V(0)=V(\frac{3}{2})=0$ so the max. is $V(\frac{1}{2})=\frac{1}{2}(2)^2=2$ ft^3